But the f^* value of any node on an optimal path is equal to $f^*(s)$, the minimal cost, and therefore $f(n') \leq f^*(s)$. Thus, we have:

> **RESULT 2**: At any time before **A*** terminates, there exists on *OPEN* a node n' that is on an optimal path from s to a goal node, with $f(n') \leq f^*(s)$.

Combining this result with our previous argument, that even the smallest f values of the nodes on *OPEN* of a nonterminating **A*** become unbounded, shows that **A*** must terminate even for infinite graphs. Thus,

> **RESULT 3**: If there is a path from s to a goal node, **A*** terminates.

RESULT 3 has an interesting corollary, namely, that any node, n, on *OPEN* with $f(n) < f^*(s)$ will eventually be selected for expansion by **A***. We leave the proof as an exercise for the reader.

Now it is a simple matter to show that **A*** is admissible. First, we note again that **A*** can either terminate by finding a goal node in step 5 or, after depleting *OPEN*, in step 3. But *OPEN* can never become empty before termination if there is a path from s to a goal node because, by RESULT 2, there will always be a node on *OPEN* (and on an optimal path). Therefore, **A*** must terminate by finding a goal node.

Next we would like to show that **A*** only terminates by finding an optimal path to a goal node. Suppose **A*** were to terminate at some goal node, t, without finding an optimal path, that is, $f(t) = g(t) > f^*(s)$. But, by RESULT 2, there existed just before termination a node, n', on *OPEN* and on an optimal path with $f(n') \leq f^*(s) < f(t)$. Thus, at this stage, **A*** would have selected n' for expansion rather than t, contradicting our supposition that **A*** terminated. Therefore, we finally have

> **RESULT 4**: Algorithm **A*** is admissible. (That is, if there is a path from s to a goal node, **A*** terminates by finding an optimal path.)

Each node selected for expansion by **A*** has an interesting property that follows directly from RESULT 2: Its f value is never greater than the cost, $f^*(s)$, of an optimal path. This result will be important to us later. To show that it is true, let n be any node selected for expansion by **A***. If n

is a goal node, we have $f(n) = f^*(s)$ by RESULT 4; so suppose n is not a goal node. Now **A*** selected n before termination, so at this time (by RESULT 2) we know that there existed on *OPEN* some node n' on an optimal path from s to a goal with $f(n') \leq f^*(s)$. If $n = n'$, our result is established. Otherwise, we know that **A*** chose to expand n rather than n'; therefore it must have been the case that

$$f(n) \leq f(n') \leq f^*(s) .$$

Therefore, we have

> **RESULT 5**: For any node n selected for expansion by **A***, $f(n) \leq f^*(s)$.

2.4.4. COMPARISON OF A* ALGORITHMS

The precision of our heuristic function h depends on the amount of heuristic knowledge it possesses about the problem domain. Clearly, using $h(n) \equiv 0$ reflects complete absence of any heuristic information about the problem, even though such an estimate is a lower bound on $h^*(n)$ and therefore leads to an admissible algorithm.

Let us compare two versions of **A***, namely, $\mathbf{A_1}$ and $\mathbf{A_2}$ using the following evaluation functions:

$$f_1(n) = g_1(n) + h_1(n)$$

and

$$f_2(n) = g_2(n) + h_2(n)$$

where h_1 and h_2 are both lower bounds on h^*. We say that algorithm $\mathbf{A_2}$ is *more informed* than algorithm $\mathbf{A_1}$ if for all nongoal nodes, n, $h_2(n) > h_1(n)$. This definition seems intuitively reasonable, since with h bounded from above by h^* for admissibility, one suspects that using larger values of h (and thus values closer to h^*) requires more accurate heuristic information.

As an example, consider the 8-puzzle solved in Figure 2.8. There we used the evaluation function $f(n) = d(n) + W(n)$. We can interpret the search process of that example as an application of **A*** with