# Lecture 26

# Rule Learning and Extraction

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Readings:

Sections 10.1-10.5, Mitchell

Section 21.4, Russell and Norvig

Section 5.4.5, Shavlik and Dietterich (Shavlik and Towell)

# Lecture Outline

- Readings: Sections 10.1-10.5, Mitchell; Section 21.4 Russell and Norvig
- Suggested Exercises: 10.1, 10.2 Mitchell
- This Week's Paper Review (Last One!)
  – "An Approach to Combining Explanation-Based and Neural Network Algorithms", Shavlik and Towell
  – Due today, 11/30/1999
- Sequential Covering Algorithms
  – Learning single rules by search
    - Beam search
    - Alternative covering methods
  – Learning rule sets
- First-Order Rules
  – Learning single first-order rules
  – FOIL: learning first-order rule sets

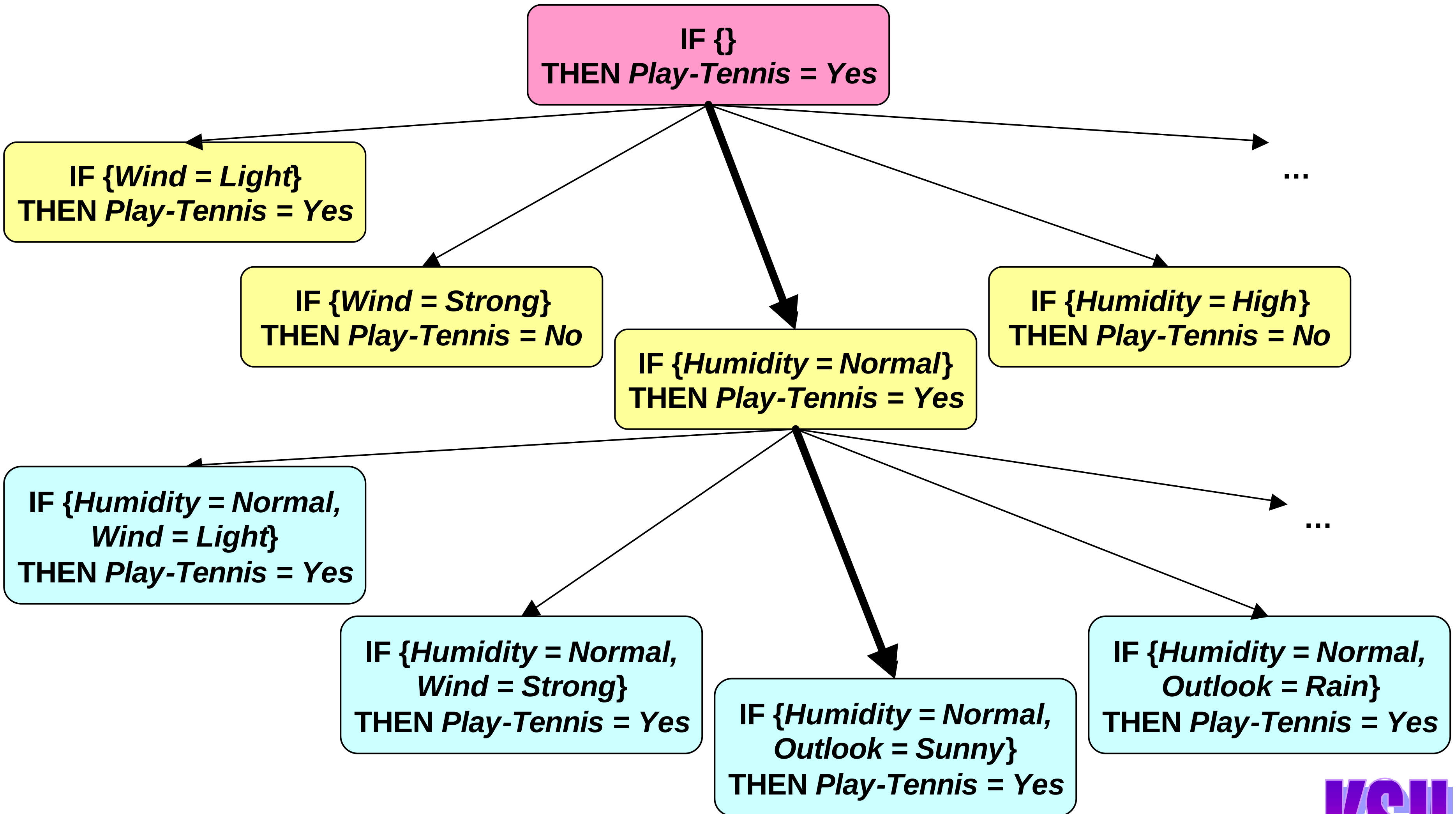
# Learning Disjunctive Sets of Rules

- **Method 1: Rule Extraction from Trees**
  - **Learn decision tree**
  - **Convert to rules**
    - **One rule per root-to-leaf path**
    - **Recall: can *post-prune* rules (drop pre-conditions to improve validation set accuracy)**
- **Method 2: Sequential Covering**
  - **Idea: greedily (sequentially) find rules that apply to (cover) instances in *D***
  - **Algorithm**
    - ***Learn one rule* with high accuracy, any coverage**
    - **Remove positive examples (of target attribute) covered by this rule**
    - **Repeat**

# Sequential Covering: Algorithm

- **Algorithm *Sequential-Covering* (*Target-Attribute*, *Attributes*, *D*, *Threshold*)**
  - ***Learned-Rules*** ? **{}**
  - ***New-Rule*** ? ***Learn-One-Rule* (*Target-Attribute*, *Attributes*, *D*)**
  - **WHILE *Performance* (*Rule*, *Examples*) > *Threshold* DO**
    - ***Learned-Rules* += *New-Rule*** **// add new rule to set**
    - ***D.Remove-Covered-By* (*New-Rule*)** **// remove examples covered by *New-Rule***
    - ***New-Rule*** ? ***Learn-One-Rule* (*Target-Attribute*, *Attributes*, *D*)**
  - ***Sort-By-Performance* (*Learned-Rules, Target-Attribute, D*)**
  - **RETURN *Learned-Rules***
- **What Does *Sequential-Covering* Do?**
  - **Learns one rule, *New-Rule***
  - **Takes out every example in *D* to which *New-Rule* applies (every <u>covered</u> example)**

# Learn-One-Rule:
# (Beam) Search for Preconditions



# Learn-One-Rule: Algorithm

- **Algorithm *Sequential-Covering* (*Target-Attribute*, *Attributes*, *D*)**
  - ***Pos*** ? ***D.Positive-Examples*()**
  - ***Neg*** ? ***D.Negative-Examples*()**
  - **WHILE NOT *Pos.Empty*() DO** **// learn new rule**
    - ***Learn-One-Rule* (*Target-Attribute*, *Attributes*, *D*)**
    - ***Learned-Rules*.*Add-Rule* (*New-Rule*)**
    - ***Pos*.*<u>Remove-Covered-By</u>* (*New-Rule*)**
  - **RETURN (*Learned-Rules*)**
- **Algorithm *Learn-One-Rule* (*Target-Attribute*, *Attributes*, *D*)**
  - ***New-Rule*** ? **most general rule possible**
  - ***New-Rule-Neg*** ? ***Neg***
  - **WHILE NOT *New-Rule-Neg.Empty*() DO** **// specialize *New-Rule***
    1. ***Candidate-Literals*** ? ***<u>Generate-Candidates</u>*()** **// *NB*: rank by *Performance*()**
    2. ***Best-Literal*** ? ***argmax***$_{L ?\ Candidate\text{-}Literals}$ ***Performance* (*<u>Specialize-Rule</u>* (*New-Rule*, *L*), *Target-Attribute*, *D*)** **// all possible new constraints**
    3. ***New-Rule.Add-Precondition* (*Best-Literal*)** **// add the best one**
    4. ***New-Rule-Neg*** ? ***New-Rule-Neg.<u>Filter-By</u>* (*New-Rule*)**
  - **RETURN (*New-Rule*)**

# Learn-One-Rule: Subtle Issues

- **How Does *Learn-One-Rule* Implement Search?**
  - Effective approach: *Learn-One-Rule* organizes *H* in same general fashion as *ID3*
  - Difference
    - Follows only *most promising branch in tree* at each step
    - Only *one attribute-value pair* (versus splitting on all possible values)
  - General to specific search (depicted in figure)
    - Problem: greedy depth-first search susceptible to local optima
    - Solution approach: <u>beam search</u> (rank by performance, always expand *k* best)
    - Easily generalizes to multi-valued target functions (how?)
- **Designing Evaluation Function to Guide Search**
  - *Performance* (*Rule*, *Target-Attribute*, *D*)
  - Possible choices
    - *Entropy* (i.e., *information gain*) as for *ID3*
    - <u>Sample accuracy</u> ($n_c$ / $n$ ? correct rule predictions / total predictions)
    - <u>*m* estimate</u>: ($n_c$ + $mp$) / ($n$ + $m$) where $m$ ? weight, $p$ ? <u>prior of rule RHS</u>

# Variants of Rule Learning Programs

- **Sequential or Simultaneous Covering of Data?**
  - **<u>Sequential</u>: isolate components of hypothesis (e.g., search for *one rule at a time*)**
  - **<u>Simultaneous</u>: whole hypothesis at once (e.g., search for *whole tree at a time*)**
- **General-to-Specific or Specific-to-General?**
  - **<u>General-to-specific</u>: add preconditions, *Find-G***
  - **<u>Specific-to-general</u>: drop preconditions, *Find-S***
- **Generate-and-Test or Example-Driven?**
  - **<u>Generate-and-test</u>: search through syntactically legal hypotheses**
  - **<u>Example-driven</u>: *Find-S*, *Candidate-Elimination*, *Cigol* (next time)**
- **Post-Pruning of Rules?**
  - **Recall (Lecture 5): very popular overfitting recovery method**
- **What Statistical Evaluation Method?**
  - **Entropy**
  - **Sample accuracy (*aka* <u>relative frequency</u>)**
  - ***m*-estimate of accuracy**



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# First-Order Rules

- **What Are First-Order Rules?**
  - **Well-formed formulas (WFFs) of first-order predicate calculus (FOPC)**
  - **Sentences of first-order logic (FOL)**
  - **Example (recursive)**
    - ***Ancestor* (*x*, *y*) ? *Parent* (*x*, *y*).**
    - ***Ancestor* (*x*, *y*) ? *Parent* (*x*, *z*) ? *Ancestor* (*z*, *y*).**
- **Components of FOPC Formulas: Quick Intro to Terminology**
  - **Constants: e.g., *John*, *Kansas*, 42**
  - **Variables: e.g., *Name*, *State*, *x***
  - **Predicates: e.g., *Father-Of*, *Greater-Than***
  - **Functions: e.g., *age*, *cosine***
  - **Term: constant, variable, or *function*(*term*)**
  - **Literals (atoms): *Predicate*(*term*) or negation (e.g., ? *Greater-Than* (*age*(*John*), 42))**
  - **Clause: disjunction of literals with implicit universal quantification**
  - **Horn clause: at most one positive literal (*H* ? ? $L_1$ ? ? $L_2$ ? … ? ? $L_n$)**

# Learning First-Order Rules

- **Why Do That?**
  - Can learn sets of rules such as
    - ***Ancestor*** **(*x*, *y*)** ? ***Parent*** **(*x*, *y*).**
    - ***Ancestor*** **(*x*, *y*)** ? ***Parent*** **(*x*, *z*)** ? ***Ancestor*** **(*z*, *y*).**
  - General-purpose (Turing-complete) programming language PROLOG
    - Programs are such sets of rules (Horn clauses)
    - Inductive logic programming (next time): kind of *program synthesis*
- **Caveat**
  - Arbitrary inference using first-order rules is semi-decidable
    - Recursive enumerable but not recursive (reduction to halting problem $L_H$)
    - Compare: resolution theorem-proving; arbitrary queries in Prolog
  - Generally, may have to restrict power
    - Inferential completeness
    - Expressive power of Horn clauses
    - *Learning part*

# First-Order Rule: Example

- **Prolog (FOPC) Rule for Classifying Web Pages**
  - **[Slattery, 1997]**
  - ***Course* (*A*)** ?
    - ***Has-Word* (*A*, *"instructor"*),**
    - **not *Has-Word* (*A*, *"good"*),**
    - ***Link-From* (*A*, *B*),**
    - ***Has-Word* (*B*, *"assign"*),**
    - **not *Link-From* (*B*, *C*).**
  - **Train: 31/31, test: 31/34**
- **How Are Such Rules Used?**
  - **Implement search-based (inferential) programs**
  - **References**
    - **Chapters 1-10, Russell and Norvig**
    - **Online resources at http://archive.comlab.ox.ac.uk/logic-prog.html**

# First-Order Inductive Learning (FOIL): Algorithm

- **Algorithm *FOIL* (*Target-Predicate*, *Predicates*, *D*)**
  - ***Pos*** ? ***D.Filter-By*(*Target-Predicate*)** **// examples for which it is true**
  - ***Neg*** ? ***D.Filter-By*(*Not* (*Target-Predicate*))** **// examples for which it is false**
  - **WHILE NOT *Pos.Empty*() DO** **// learn new rule**
    - ***Learn-One-First-Order-Rule* (*Target-Predicate*, *Predicates*, *D*)**
    - ***Learned-Rules*.*Add-Rule* (*New-Rule*)**
    - ***Pos*.*Remove-Covered-By* (*New-Rule*)**
  - **RETURN (*Learned-Rules*)**
- **Algorithm *Learn-One-First-Order-Rule* (*Target-Predicate*, *Predicate*, *D*)**
  - ***New-Rule*** ? **the rule that predicts *Target-Predicate* with no preconditions**
  - ***New-Rule-Neg*** ? ***Neg***
  - **WHILE NOT *New-Rule-Neg.Empty*() DO** **// specialize *New-Rule***
    1. ***Candidate-Literals*** ? ***Generate-Candidates*()** **// based on *Predicates***
    2. ***Best-Literal*** ? ***argmax***$_{L ? \text{ Candidate-Literals}}$ ***FOIL-Gain* (*L, New-Rule, Target-Predicate, D*)** **// all possible new literals**
    3. ***New-Rule.Add-Precondition* (*Best-Literal*)** **// add the best one**
    4. ***New-Rule-Neg*** ? ***New-Rule-Neg,Filter-By* (*New-Rule*)**
  - **RETURN (*New-Rule*)**

# Specializing Rules in FOIL

- **Learning Rule:** $P(x_1, x_2, \ldots, x_k)$ ? $L_1$ ? $L_2$ ? … ? $L_n$.
- **Candidate Specializations**
  - Add new literal to get more specific Horn clause
  - Form of literal
    - $Q(v_1, v_2, \ldots, v_r)$, where at least one of the $v_i$ in the created literal must already exist as a variable in the rule
    - $Equal(x_j, x_k)$, where $x_j$ and $x_k$ are variables already present in the rule
    - The negation of either of the above forms of literals



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# Information Gain in FOIL

- **Function *FOIL-Gain* (*L*, *R*, *Target-Predicate*, *D*)**

$$\textit{Foil-Gain} \equiv t\left(\lg\left(\frac{p_1}{p_1 + n_1}\right) - \lg\left(\frac{p_0}{p_0 + n_0}\right)\right)$$

- **Where**
  - $L \equiv$ **candidate predicate to add to rule *R***
  - $p_0 \equiv$ **number of positive bindings of *R***
  - $n_0 \equiv$ **number of negative bindings of *R***
  - $p_1 \equiv$ **number of positive bindings of *R* + *L***
  - $n_1 \equiv$ **number of negative bindings of *R* + *L***
  - $t \equiv$ **number of positive bindings of *R* also covered by *R* + *L***
- **Note**
  - **$-\lg(p_0 / p_0 + n_0)$ is optimal number of bits to indicate the class of a positive binding covered by *R***
  - **Compare: entropy (information gain) measure in *ID3***

# FOIL: Learning Recursive Rule Sets

- **Recursive Rules**
  - So far: ignored possibility of recursive WFFs
    - New literals added to rule body could refer to target predicate itself
    - i.e., predicate occurs in rule head
  - Example
    - ***Ancestor*** **(*x*, *y*)** ? ***Parent*** **(*x*, *z*)** ? ***Ancestor*** **(*z*, *y*).**
    - Rule: IF ***Parent*** **(*x*, *z*)** ? ***Ancestor*** **(*z*, *y*)** THEN ***Ancestor*** **(*x*, *y*)**
- **Learning Recursive Rules from Relations**
  - Given: appropriate set of training examples
  - Can learn using *FOIL*-based search
    - Requirement: *Ancestor* ? *Predicates* (symbol is member of candidate set)
    - Recursive rules still have to outscore competing candidates at *FOIL-Gain*
  - *NB*: how to ensure termination? (<u>well-founded ordering</u>, i.e., no infinite recursion)
  - [Quinlan, 1990; Cameron-Jones and Quinlan, 1993]

# FOIL: Summary

- Extends *Sequential-Covering* Algorithm
  - Handles case of learning first-order rules similar to Horn clauses
  - Result: *more powerful rules for performance element* (automated reasoning)
- General-to-Specific Search
  - Adds literals (predicates and negations over functions, variables, constants)
  - Can learn sets of recursive rules
    - Caveat: might learn infinitely recursive rule sets
    - Has been shown to successfully induce recursive rules in some cases
- Overfitting
  - If no noise, might keep adding new literals until rule covers *no negative examples*
  - Solution approach: tradeoff (heuristic evaluation function on rules)
    - Accuracy, coverage, complexity
    - *FOIL-Gain*: an MDL function
    - Overfitting recovery in *FOIL*: post-pruning

# Terminology

- **Disjunctive Rules**
  - Learning single rules by search
    - Beam search: type of heuristic search that maintains constant-width frontier
  - Learning rule sets
    - Sequential covering versus simultaneous covering
- **First-Order Rules**
  - Units of first-order predicate calculus (FOPC): constants, variables, predicates, functions, terms, literals (atoms), well-formed fomulas (wffs, clauses)
  - FOPC quantifiers: universal (? ), existential (? )
  - Horn clauses
    - Sentences of Prolog (clauses with ? 1 positive literal)
    - Of the form: $H$ ? ? $L_1$ ? ? $L_2$ ? … ? ? $L_n$ (implicit ? ), Prolog form $H$ :- $L_1$, $L_2$, …, $L_n$.
  - FOIL: algorithm for learning Horn clauses (including recursive rule sets)

# Summary Points

- Learning Rules from Data
- Sequential Covering Algorithms
  - Learning single rules by search
    - Beam search
    - Alternative covering methods
  - Learning rule sets
- First-Order Rules
  - Learning single first-order rules
    - Representation: first-order Horn clauses
    - Extending *Sequential-Covering* and Learn-One-Rule: variables in rule preconditions
  - FOIL: learning first-order rule sets
    - Idea: inducing logical rules from observed relations
    - Guiding search in FOIL
    - Learning recursive rule sets
- Next Time: Inductive Logic Programming (ILP)