

Lecture 24 of 41

Collision Handling Part 1 of 2: Separating Axes, Oriented Bounding Boxes

William H. Hsu

Department of Computing and Information Sciences, KSU

KSOL course pages: <http://bit.ly/hGvXIH> / <http://bit.ly/eVizrE>

Public mirror web site: <http://www.kddresearch.org/Courses/CIS636>

Instructor home page: <http://www.cis.ksu.edu/~bhsu>

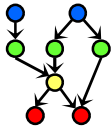
Readings:

Today: §2.4.3, 8.1, Eberly 2^e – see <http://bit.ly/ieUq45>; **GL handout**

Next class: Chapter 6, esp. §6.1, Eberly 2^e

Wikipedia, *Collision Detection*: <http://bit.ly/14rFzG>

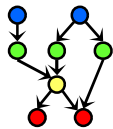




Lecture Outline

- Reading for Last Class: Chapter 10, 13, §17.3 – 17.5, Eberly 2^e
- Reading for Today: §2.4.3, 8.1, Eberly 2^e, **GL handout**
- Reading for Next Class: Chapter 6, Esp. §6.1, Eberly 2^e
- Last Time: Quaternions Concluded
 - ✦ How quaternions work – properties, matrix equivalence, arithmetic
 - ✦ Composing rotations by quaternion multiplication
 - ✦ Incremental rotation and error issues
- Videos 4: Modeling & Simulation, Visualization; VR/VE/VA/AR
 - ✦ Virtual reality, environments, artifacts (VR/VE/VA); augmented reality
 - ✦ Relationship among visualization, simulation, & animation
- Today: Collision Detection Part 1 of 2
 - ✦ Test-intersection queries vs. find-intersection queries
 - ✦ Static: stationary objects (both not moving)
 - ✦ Dynamic: moving objects (one or both)
 - ✦ Distance vs. intersection methods





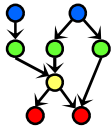
Where We Are

21	Lab 4a: Animation Basics	Flash animation handout
22	Animation 2: Rotations; Dynamics, Kinematics	Chapter 17, esp. §17.1 – 17.2
23	Demos 4: Modeling & Simulation, Rotations	Chapter 10 ¹ , 13 ² , §17.3 – 17.5
24	Collisions 1: axes, OBBs, Lab 4b	§2.4.3, 8.1, GL handout
25	Spatial Sorting: Binary Space Partitioning	Chapter 6, esp. §6.1
26	Demos 5: More CGA; Picking; HW/Exam	Chapter 7 ² ; § 8.4
27	Lab 5a: Interaction Handling	§ 8.3 – 8.4; 4.2, 5.0, 5.6, 9.1
28	Collisions 2: Dynamic, Particle Systems	§ 9.1, particle system handout
	Exam 2 review; Hour Exam 2 (evening)	Chapters 5 – 6, 7 ² – 8, 12, 17
29	Lab 5b: Particle Systems	Particle system handout
30	Animation 3: Control & IK	§ 5.3, CGA handout
31	Ray Tracing 1: intersections, ray trees	Chapter 14
32	Lab 6a: Ray Tracing Basics with POV-Ray	RT handout
33	Ray Tracing 2: advanced topic survey	Chapter 15, RT handout
34	Visualization 1: Data (Quantities & Evidence)	Tufte handout (1)
35	Lab 6b: More Ray Tracing	RT handout
36	Visualization 2: Objects	Tufte handout (2 & 4)
37	Color Basics; Term Project Prep	Color handout
38	Lab 7: Fractals & Terrain Generation	Fractals/Terrain handout
39	Visualization 3: Processes; Final Review 1	Tufte handout (3)
40	Project presentations 1; Final Review 2	–
41	Project presentations 2	–
	Final Exam	Ch. 1 – 8, 10 – 15, 17, 20

Lightly-shaded entries denote the due date of a written problem set; heavily-shaded entries, that of a machine problem (programming assignment); blue-shaded entries, that of a paper review; and the green-shaded entry, that of the term project.

Green, blue and red letters denote exam review, exam, and exam solution review dates.





Acknowledgements: Quaternions, Collision Handling



Rick Parent

Professor

Department of Computer Science and Engineering

Ohio State University

<http://www.cse.ohio-state.edu/~parent/>



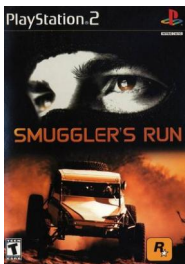
David C. Brogan

Visiting Assistant Professor, Computer Science Department, University of Virginia

<http://www.cs.virginia.edu/~dbrogan/>

Susquehanna International Group (SIG)

<http://www.sig.com>



Steve Rotenberg

Visiting Lecturer

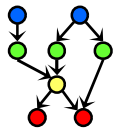
Graphics Lab

University of California – San Diego

CEO/Chief Scientist, PixelActive

<http://graphics.ucsd.edu>

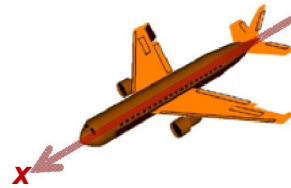




Review [1]: Fixed Angles & Euler Angles

Rotation about x axis
(Roll)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) & 0 \\ 0 & \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Rotation about y axis
(Pitch)

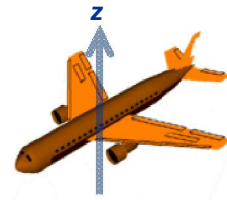
$$\begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



106 Wikipedia,
Flight Dynamics
[/bit.ly/gVaQCX](http://bit.ly/gVaQCX)

Rotation about z axis
(Yaw)

$$\begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

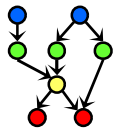


Adapted from slides ♥ 2007 – 2011 R. Parent, Ohio State University

CSE 682 (Computer Animation), <http://bit.ly/feUESy>

CSE 683/684A (Computer Animation Algorithms & Techniques), <http://bit.ly/f8Myky>





Review [2]: Axis-Angle to Quaternion Conversion

A quaternion is a 4-D unit vector $q = [x \ y \ z \ w]$

- It lies on the unit hypersphere $x^2 + y^2 + z^2 + w^2 = 1$

For rotation about (unit) axis v by angle θ

- vector part = $(\sin \theta/2) \ v = [x \ y \ z]$
- scalar part = $(\cos \theta/2) = w$
- $(\sin(\theta/2) \ n_x, \sin(\theta/2) \ n_y, \sin(\theta/2) \ n_z, \cos(\theta/2))$

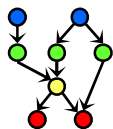
Only a unit quaternion encodes a rotation - normalize

Adapted from slides ♥ 2000 – 2004 D. Brogan, University of Virginia
CS 445/645, Introduction to Computer Graphics, <http://bit.ly/h9AHRg>



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Review [3]: Quaternion to RM Conversion

Rotation matrix corresponding to a quaternion:

$$[x \ y \ z \ w] = \begin{bmatrix} 1 - 2y^2 - 2z^2 & 2xy + 2wz & 2xz - 2wy \\ 2xy - 2wz & 1 - 2x^2 - 2z^2 & 2yz + 2wx \\ 2xz + 2wy & 2yz - 2wx & 1 - 2x^2 - 2y^2 \end{bmatrix}$$

Quaternion Multiplication

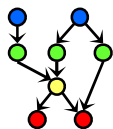
- $q_1 * q_2 = [v_1, w_1] * [v_2, w_2] = [(w_1v_2 + w_2v_1 + (v_1 \times v_2)), w_1w_2 - v_1 \cdot v_2]$
- quaternion * quaternion = quaternion
- this satisfies requirements for mathematical *group*
- Rotating object twice according to two different quaternions is equivalent to one rotation according to product of two quaternions

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CS 445/645, Introduction to Computer Graphics, <http://bit.ly/h9AHRg>



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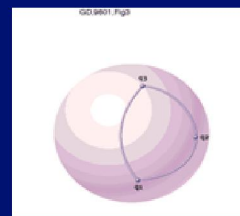




Review [4]: Advantage – Interpolation

Biggest advantage of quaternions

- Interpolation
- Cannot linearly interpolate between two quaternions because it would speed up in middle
- Instead, Spherical Linear Interpolation, `slerp()`
- Used by modern video games for third-person perspective
- Why?



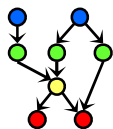
Hint: see <http://youtu.be/-jBKKV2V8eU>

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CS 445/645, Introduction to Computer Graphics, <http://bit.ly/h9AHRg>



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Interpolating Quaternions [1]: Lerp

- If we want to do a linear interpolation between two points **a** and **b** in normal space

$$\text{Lerp}(t, \mathbf{a}, \mathbf{b}) = (1-t)\mathbf{a} + (t)\mathbf{b}$$

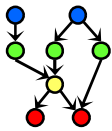
where t ranges from 0 to 1

- Note that the Lerp operation can be thought of as a weighted average (convex)
- We could also write it in its additive blend form:

$$\text{Lerp}(t, \mathbf{a}, \mathbf{b}) = \mathbf{a} + t(\mathbf{b} - \mathbf{a})$$

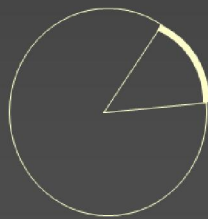
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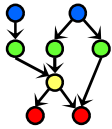
Interpolating Quaternions [2]: Slerp

- If we want to interpolate between two points on a sphere (or hypersphere), we don't just want to Lerp between them
- Instead, we will travel across the surface of the sphere by following a 'great arc'



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Interpolating Quaternions [3]: Slerp Optimization

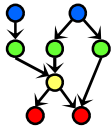
- Remember that there are two redundant vectors in quaternion space for every unique orientation in 3D space
- What is the difference between:

$\text{Slerp}(t, \mathbf{a}, \mathbf{b})$ and $\text{Slerp}(t, -\mathbf{a}, \mathbf{b})$?

- One of these will travel less than 90 degrees while the other will travel more than 90 degrees across the sphere
- This corresponds to rotating the 'short way' or the 'long way'
- Usually, we want to take the short way, so we negate one of them if their dot product is < 0

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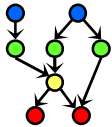
Review [5]: Dynamics & Kinematics

- **Dynamics: Study of Motion & Changes in Motion**
 - ✱ Forward: model forces over time to find state, e.g.,
 - Given: initial position p_0 , velocity v_0 , gravitational constants
 - Calculate: position p_t at time t
 - ✱ Inverse: given state and constraints, calculate forces, e.g.,
 - Given: *desired* position p_t at time t , gravitational constants
 - Calculate: position p_0 , velocity v_0 needed
 - ✱ Wikipedia: <http://bit.ly/hH43dX> (see also: “Analytical dynamics”)
 - ✱ For non-particle objects: rigid-body dynamics (<http://bit.ly/dLvejg>)
- **Kinematics: Study of Motion without Regard to Causative Forces**
 - ✱ Modeling systems – e.g., articulated figure
 - ✱ Forward: from angles to position (<http://bit.ly/eh2d1c>)
 - ✱ Inverse: finding angles given desired position (<http://bit.ly/hsyTb0>)
 - ✱ Wikipedia: <http://bit.ly/hr8r2u>



Forward Kinematics
© 2009 Wikipedia



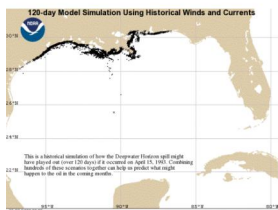
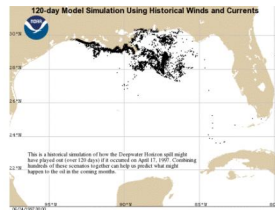
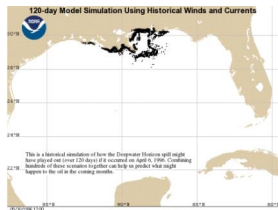


Review [6]: Visualization & Simulation

Deepwater Horizon Oil Spill (20 Apr 2010)

<http://bit.ly/9QHax4>

120-day images © 2010 NOAA, <http://1.usa.gov/c02xuQ>



120-day simulation using
15 Apr 1993 weather conditions



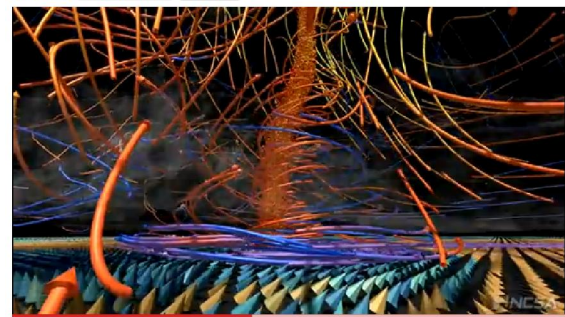
132-day simulation using
2010 conditions
© 2010 National Center for
Supercomputing
Applications (NCSA)

http://youtu.be/pE-1G_476nA

YouTube

Visualization Of An F3 Tornado Within A Supercell Thunderstorm Sim

djxatlanta 1,354 videos Subscribe



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Uploaded by djxatlanta on Jan 15, 2010

Scientists used pre-storm conditions from an observed F4 tornado in South

7 likes, 0 dislikes

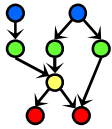
Wilhelmson et al. (2004)

<http://youtu.be/EgumU0Ns1YI>

<http://avl.ncsa.illinois.edu>

<http://bit.ly/eA8PXN>





Review [7]: Virtual Reality (VR)

- Virtual Reality: Computer-Simulated Environments
- Physical Presence: Real & Imaginary
- Hardware: User Interface
 - * Head-mounted display (HMD), gloves – see PopOptics goggles (left)
 - * VR glasses, wand, etc. – see NCSA CAVE (right)

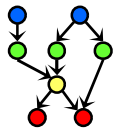


Virtual Reality, Wikipedia:
<http://bit.ly/fAvNeP>
 Image © 2007 National Air & Space Museum



CAVE (Cave Automatic Virtual Environment)
 Image © 2009 D. Pape
 HowStuffWorks article: <http://bit.ly/feQxNK>
 © 2009 J. Strickland
 Wikipedia: <http://bit.ly/dKNEnU>





Review [8]: Virtual Environments (VE)

- Virtual Environment: Part of Virtual Reality Experience
- Other Parts
 - ✦ Virtual artifacts (VA): simulated objects – <http://bit.ly/hskSyX>
 - ✦ Intelligent agents, artificial & real – <http://bit.ly/y2gQk>

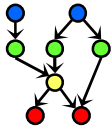


Experia © 2006 M. Vanderbeeken et al., <http://bit.ly/hzfAQx>
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We Are Arcade © 2011 D. Grossett et al., <http://bit.ly/ftALjU>
World of Warcraft: Cataclysm review © 2011
 J. Greer, <http://bit.ly/eENHXt>
World of Warcraft © 2001 – 2011
 Blizzard Entertainment, Inc.,
<http://bit.ly/2qvPYF>





Review [9]: Augmented Reality (AR)

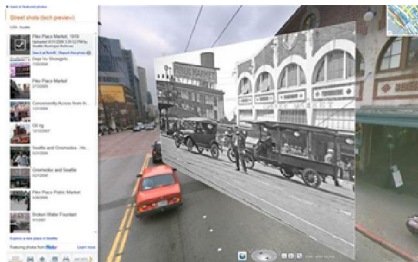
- **Augmented Reality: Computer-Generated (CG) Sensory Overlay**
- **Added to Physical, Real-World Environment**



"40 Best Augmented Reality iPhone Applications",
© 2010 iPhoneNess.com, <http://bit.ly/2qT35y>
MyNav © 2010 Winfield & Co. <http://bit.ly/dLTir7>

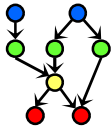


Wikipedia, Google Goggles:
<http://bit.ly/gRRMLS>



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Acknowledgements: Intersections, Containment – Eberly 1^e

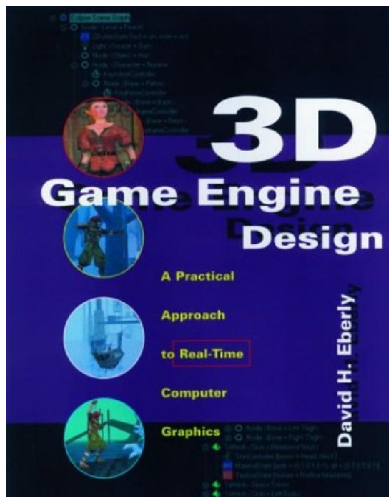
David H. Eberly

Chief Technology Officer

Geometric Tools, LLC

<http://www.geometrictools.com>

<http://bit.ly/enKbfs>



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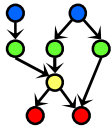
Today's material:

- View Frustum clipping
 - §2.4.3, p. 70 – 77, 2^e
 - §3.4.3, p. 93 – 99, & §3.7.2, p. 133 – 136, 1^e
- Collision detection: separating axes
 - §8.1, p. 393 – 443, 2^e
 - §6.4. p. 203 – 214, 1^e

Later:

- Distance methods
 - Chapter 14, p. 639 – 679, 2^e
 - §2.6, p. 38 – 77, 1^e
- Intersection methods
 - Chapter 15, p. 681 – 717, 2^e
 - §6.2 – 6.5, p. 188 – 243, 1^e





View Frustum Clipping: Triangle Splitting

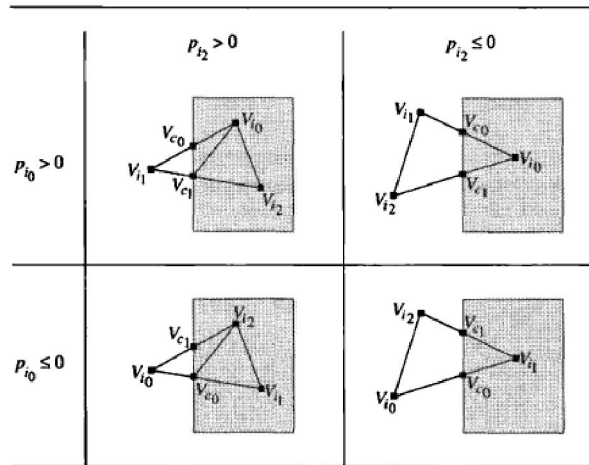
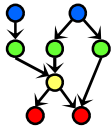


Figure 3.4 Four configurations for triangle splitting. Only the triangles in the shaded region are important, so the quadrilaterals outside are not split.

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Collision Handling: Detection vs. Response

■ Collision Detection

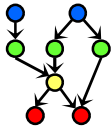
- Collision detection is a geometric problem
- Given two moving objects defined in an initial and final configuration, determine if they intersected at some point between the two states

■ Collision Response

- The response to collisions is the actual physics problem of determining the unknown forces (or impulses) of the collision

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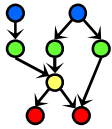


Collision Detection [1]: Technical Problem Defined

- 'Collision detection' is really a geometric intersection detection problem
- Main subjects
 - Intersection testing (triangles, spheres, lines...)
 - Optimization structures (octree, BSP...)
 - Pair reduction (reducing N^2 object pair testing)

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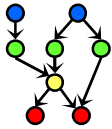


Collision Detection [2]: Intersections – Testing vs. Finding

- General goals: given two objects with current and previous orientations specified, determine if, where, and when the two objects intersect
- Alternative: given two objects with only current orientations, determine if they intersect
- Sometimes, we need to find all intersections. Other times, we just want the first one. Sometimes, we just need to know if the two objects intersect and don't need the actual intersection data.

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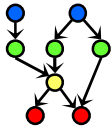


Collision Detection [3]: Queries – Test- vs. Find-Intersection

- **Test-Intersection: Determine If Objects Intersect**
 - * Static: test whether they do at given instant
 - * Dynamic: test whether they intersect at any point along trajectories
- **Find-Intersection: Determine Intersection (or Contact) Set of Objects**
 - * Static: intersection set (compare: $A \cap B$)
 - * Dynamic: contact time (interval of overlap), sets (depends on time)

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Collision Detection [3]: Queries – Distance vs. Intersection

- **Distance-Based**

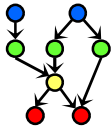
- * Parametric representation of object boundaries/interiors
- * Want: closest points on two objects (to see whether they intersect)
- * Use: constrained minimization to solve for closest points

- **Intersection-Based**

- * Also uses parametric representation
- * Want: overlapping subset of interior of two objects
- * General approach: equate objects, solve for parameters
- * Use one of two kinds of solution methods
 - Analytical (when feasible to solve exactly – e.g., OBBs)
 - Numerical (approximate region of overlap)
- * Solving for parameters in equation
- * Harder to compute than distance-based; use only when needed

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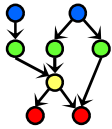


Collision Detection [4]: Primitives

- We often deal with various different ‘primitives’ that we describe our geometry with. Objects are constructed from these primitives
- Examples
 - Triangles
 - Spheres
 - Cylinders
 - AABB = axis aligned bounding box
 - OBB = oriented bounding box
- At the heart of the intersection testing are various primitive-primitive tests

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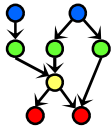


Collision Detection [5]: Particle Collisions

- For today, we will mainly be concerned with the problem of testing if particles collide with solid objects
- A particle can be treated as a line segment from its previous position to its current position
- If we are colliding against static objects, then we just need to test if the line segment intersects the object
- Colliding against moving objects requires some additional modifications that we will also look at

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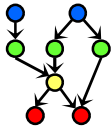


Collision Detection [6]: Code – Basic Components

```
class Segment {  
    Vector3 A,B;  
};  
  
class Intersection {  
    Vector3 Position;  
    Vector3 Normal;  
    Material *Mtl; (Mtl can contain info about  
                    elasticity, friction, etc)  
};
```

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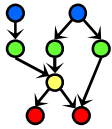


Collision Detection [7]: Code – Primitives

```
class Primitive {  
    virtual bool TestSegment(const Segment &s,  
        Intersection &i);  
};  
  
class Sphere:public Primitive...  
class Triangle:public Primitive...  
class Cylinder:public Primitive...
```

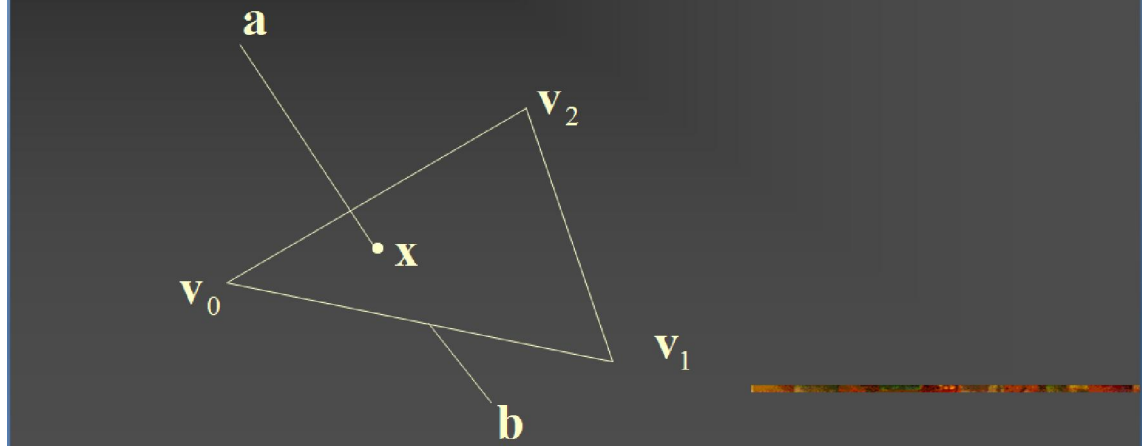
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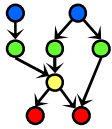
Collision Detection [8]: Segment vs. Triangle – Query

- Does segment **ab** intersect triangle $v_0v_1v_2$?



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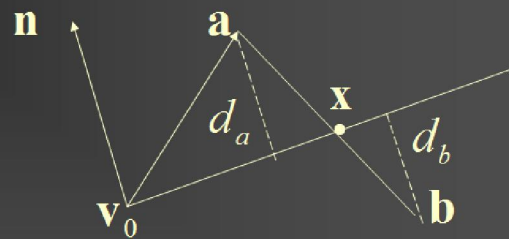


Collision Detection [9]: Segment vs. Triangle – Solution

- First, compute signed distances of a and b to plane

$$d_a = (\mathbf{a} - \mathbf{v}_0) \cdot \mathbf{n}$$

$$d_b = (\mathbf{b} - \mathbf{v}_0) \cdot \mathbf{n}$$

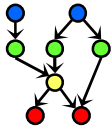


- Reject if both are above or both are below triangle
- Otherwise, find intersection point \mathbf{x}

$$\mathbf{x} = \frac{d_a \mathbf{b} - d_b \mathbf{a}}{d_a - d_b}$$

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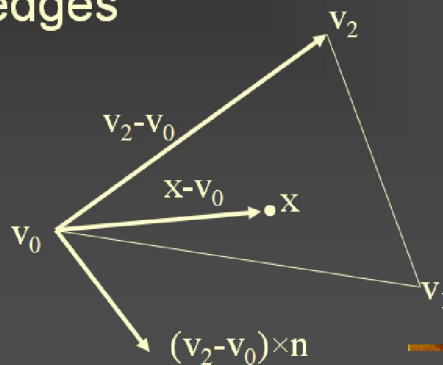


Collision Detection [10]: Segment vs. Triangle – Point Test

- Is point x inside the triangle?

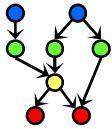
$$(x-v_0) \cdot ((v_2-v_0) \times n) > 0$$

- Test all 3 edges



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Collision Detection [11]: Faster Triangle – Point Containment

- Reduce to 2D: remove smallest dimension
- Compute barycentric coordinates

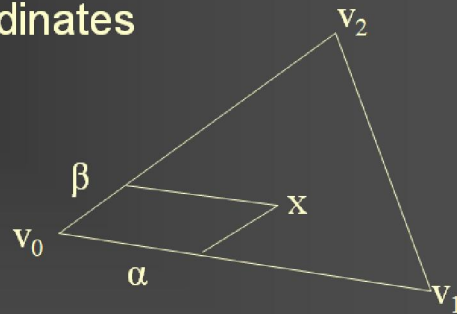
$$\mathbf{x}' = \mathbf{x} - \mathbf{v}_0$$

$$\mathbf{e}_1 = \mathbf{v}_1 - \mathbf{v}_0$$

$$\mathbf{e}_2 = \mathbf{v}_2 - \mathbf{v}_0$$

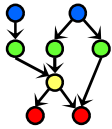
$$\alpha = (\mathbf{x}' \times \mathbf{e}_2) / (\mathbf{e}_1 \times \mathbf{e}_2)$$

$$\beta = (\mathbf{x}' \times \mathbf{e}_1) / (\mathbf{e}_1 \times \mathbf{e}_2)$$
- Reject if $\alpha < 0$, $\beta < 0$ or $\alpha + \beta > 1$



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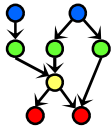


Collision Detection [12]: Segment vs. Mesh

- To test a line segment against a mesh of triangles, simply test the segment against each triangle
- Sometimes, we are interested in only the 'first' hit along the segment from **a** to **b**. Other times, we want all intersections. Still other times, we just need any intersection.
- Testing against lots of triangles in a large mesh can be time consuming. We will look at ways to optimize this later

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Collision Detection [13]: Segment vs. Moving Mesh

- M_0 is the object's matrix at time t_0
- M_1 is the matrix at time t_1
- Compute delta matrix:

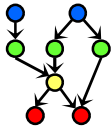
$$M_1 = M_0 \cdot M_\Delta$$

$$M_\Delta = M_0^{-1} \cdot M_1$$
- Transform a by M_Δ

$$a' = a \cdot M_\Delta$$
- Test segment $a'b$ against object with matrix M_1

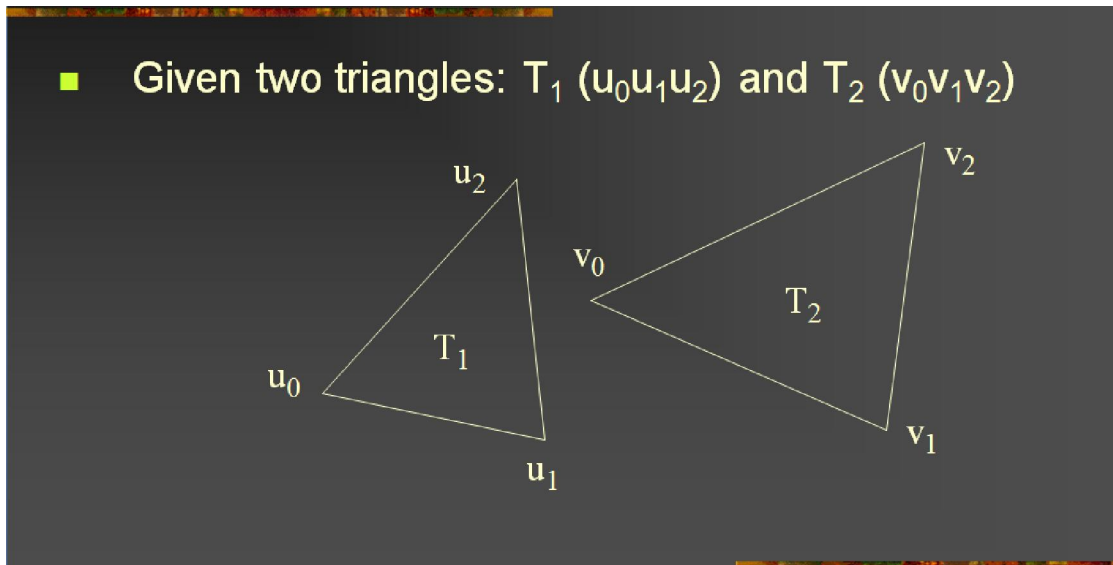
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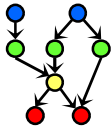
Collision Detection [14]: Triangles vs. Triangle – Query

- Given two triangles: $T_1 (u_0 u_1 u_2)$ and $T_2 (v_0 v_1 v_2)$



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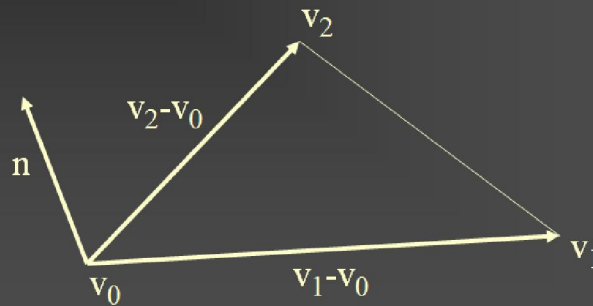


Collision Detection [15]: Triangles vs. Triangle – Plane Equations

Step 1: Compute plane equations

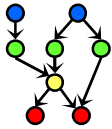
$$\mathbf{n}_2 = (\mathbf{v}_1 - \mathbf{v}_0) \times (\mathbf{v}_2 - \mathbf{v}_0)$$

$$d_2 = -\mathbf{n}_2 \cdot \mathbf{v}_0$$



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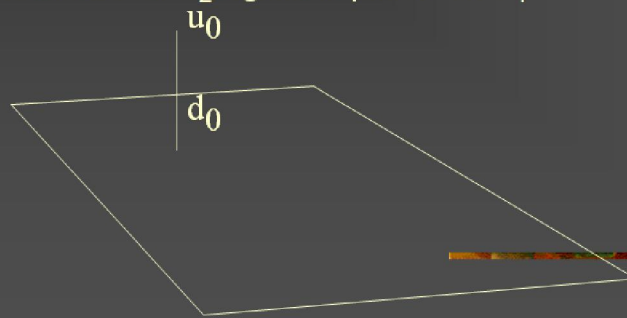


Collision Detection [16]: Triangles vs. Triangle – Distances

- Step 2: Compute signed distances of T_1 vertices to plane of T_2 :

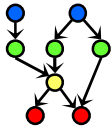
$$d_i = n_2 \cdot u_i + d_2 \quad (i=0,1,2)$$

- Reject if all $d_i < 0$ or all $d_i > 0$
- Repeat for vertices of T_2 against plane of T_1



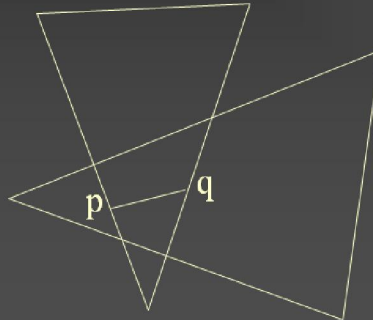
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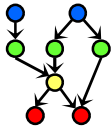
Collision Detection [17]: Triangles vs. Triangle – Intersection

- Step 3: Find intersection points
- Step 4: Determine if segment pq is inside triangle or intersects triangle edge



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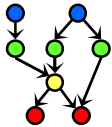


Collision Detection [18]: Mesh vs. Mesh – Kinds of Collisions

- Geometry: points, edges, faces
- Collisions: p2p, p2e, p2f, e2e, e2f, f2f
- Relevant ones: p2f, e2e (point to face & edge to edge)
- Multiple simultaneous collisions

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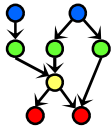


Collision Detection [19]: Moving Mesh vs. Moving Mesh

- Two options: 'point sample' and 'extrusion'
- Point sample:
 - If objects intersect at final positions, do a binary search backwards to find the time when they first hit and compute the intersection
 - This approach can tend to miss thin objects
- Extrusion:
 - Test '4-dimensional' extrusions of objects
 - In practice, this can be done using only 3D math

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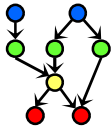


Collision Detection [20]: Moving Meshes: Extrusion

- Use 'delta matrix' trick to simplify problem so that one mesh is moving and one is static
- Test moving vertices against static faces (and the opposite, using the other delta matrix)
- Test moving edges against static edges (moving edges form a quad (two triangles))

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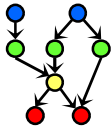


Collision Detection [21]: Intersection Issues

- Performance
- Memory
- Accuracy
- Floating point precision

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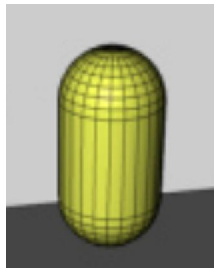


Static Intersection [1]: Sphere-Swept Volumes

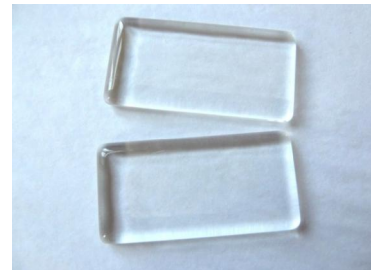
- **Sphere**
 - ✦ Locus of points in 3-D equidistant from center point
 - ✦ Rotational sweep of circle (hollow sphere) or disc (solid ball)
 - ✦ “Null” sweep of sphere (invariant under rotation, translation by 0)
- **Capsule: Translational Sweep of Sphere Along Line Segment**
- **Lozenge: Sweep of Sphere Across Rectangle**



Wikipedia: Sphere
<http://bit.ly/9OWjQi>
 Image © 2008 ClipArtOf.com
<http://bit.ly/eKhE2f>



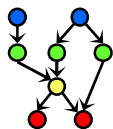
Capsule
 Image © 2007 Remotion Wiki
<http://bit.ly/huEzNW>



Lozenge
 Image © 2011 Jasmin Studio Crafts
<http://bit.ly/euEopw>

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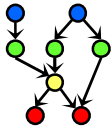
Static Intersection [2]: Distance Calculators

Table 6.1 Relationship between sphere-swept volumes and distance calculators (*pnt*, point; *seg*, line segment; *rct*, rectangle).

	<i>Sphere</i>	<i>Capsule</i>	<i>Lozenge</i>
<i>Sphere</i>	<code>dist(pnt,pnt)</code>	<code>dist(pnt,seg)</code>	<code>dist(pnt,rct)</code>
<i>Capsule</i>	<code>dist(seg,pnt)</code>	<code>dist(seg,seg)</code>	<code>dist(seg,rct)</code>
<i>Lozenge</i>	<code>dist(rct,pnt)</code>	<code>dist(rct,seg)</code>	<code>dist(rct,rct)</code>

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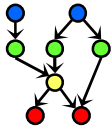
Dynamic Intersection [1]: One Moving Object

Table 6.6 Relationship between sphere-swept volumes and distance calculators when the second object is moving (*pnt*, point; *seg*, line segment; *rct*, rectangle; *pgm*, parallelogram; *ppd*, parallelepiped; *hex*, hexagon).

	Dynamic		
	<i>Sphere</i>	<i>Capsule</i>	<i>Lozenge</i>
<i>Static</i>			
Sphere	$\text{dist}(\text{pnt}, \{\text{pnt}, \text{seg}\})$	$\text{dist}(\text{pnt}, \{\text{seg}, \text{pgm}\})$	$\text{dist}(\text{pnt}, \{\text{rct}, \text{hex}, \text{ppd}\})$
Capsule	$\text{dist}(\text{seg}, \{\text{pnt}, \text{seg}\})$	$\text{dist}(\text{seg}, \{\text{seg}, \text{pgm}\})$	$\text{dist}(\text{seg}, \{\text{rct}, \text{hex}, \text{ppd}\})$
Lozenge	$\text{dist}(\text{rct}, \{\text{pnt}, \text{seg}\})$	$\text{dist}(\text{rct}, \{\text{seg}, \text{pgm}\})$	$\text{dist}(\text{rct}, \{\text{rct}, \text{hex}, \text{ppd}\})$

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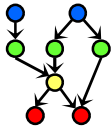
Dynamic Intersection [2]: Two Moving Objects – Separating Axes

Table 6.7 Values for R , R_0 , and R_1 for the separating axis test $R > R_0 + R_1$ for two boxes in the direction of motion.

\vec{L}	R_0	R_1	R
$\vec{W} \times \vec{A}_0$	$a_1 \alpha_2 + a_2 \alpha_1 $	$\sum_{i=0}^2 b_i c_{1i}\alpha_2 - c_{2i}\alpha_1 $	$ \vec{A}_0 \cdot \vec{W} \times \vec{D} $
$\vec{W} \times \vec{A}_1$	$a_0 \alpha_2 + a_2 \alpha_0 $	$\sum_{i=0}^2 b_i c_{0i}\alpha_2 - c_{2i}\alpha_0 $	$ \vec{A}_1 \cdot \vec{W} \times \vec{D} $
$\vec{W} \times \vec{A}_2$	$a_0 \alpha_1 + a_1 \alpha_0 $	$\sum_{i=0}^2 b_i c_{0i}\alpha_1 - c_{1i}\alpha_0 $	$ \vec{A}_2 \cdot \vec{W} \times \vec{D} $
$\vec{W} \times \vec{B}_0$	$\sum_{i=0}^2 a_i c_{i1}\beta_2 - c_{i2}\beta_1 $	$b_1 \beta_2 + b_2 \beta_1 $	$ \vec{B}_0 \cdot \vec{W} \times \vec{D} $
$\vec{W} \times \vec{B}_1$	$\sum_{i=0}^2 a_i c_{i0}\beta_2 - c_{i2}\beta_0 $	$b_0 \beta_2 + b_2 \beta_0 $	$ \vec{B}_1 \cdot \vec{W} \times \vec{D} $
$\vec{W} \times \vec{B}_2$	$\sum_{i=0}^2 a_i c_{i0}\beta_1 - c_{i1}\beta_0 $	$b_0 \beta_1 + b_1 \beta_0 $	$ \vec{B}_2 \cdot \vec{W} \times \vec{D} $

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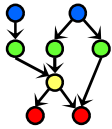


Preview: Collision Response & Optimization

- **What Happens After Collision Is Detected?**
 - * Contact & application of force vs. impact & impulse
 - * Compression: deformation of solid
 - * Restitution: springing back of solid
 - * Friction?
 - * Secondary collisions due to changes in trajectories
 - * Bouncing?
- **Optimization**
 - * Spatial partitioning: bounding volume hierarchies (BVHs) revisited
 - Binary space partitioning (BSP) trees
 - *k*-d trees
 - Quadtrees & octrees
 - * Volume graphics: uniform grids and data parallelism

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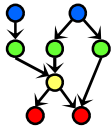




Summary

- Reading for Last Class: Chapter 10, 13, §17.3 – 17.5, Eberly 2^e
- Reading for Today: §2.4.3, 8.1, Eberly 2^e, **GL handout**
- Reading for Next Class: Chapter 6, Esp. §6.1, Eberly 2^e
- Last Time: Quaternions Concluded
 - * How quaternions work – properties, matrix equivalence, arithmetic
 - * Composing rotations by quaternion multiplication
 - * Incremental rotation and error issues
- Review: Virtual Reality & Virtual Environments; Augmented Reality
- Today: Collision Detection Part 1 of 2
 - * Static: stationary objects (both not moving)
 - * Dynamic: moving objects (one or both)
 - * Test-intersection queries vs. find-intersection queries
 - * Distance vs. intersection methods





Terminology

- **Visualization** – Communicating with Images, Diagrams, Animations
- **VR, VE, VA, AR**
 - * **Virtual Reality**: computer-simulated environments, objects
 - * **Virtual Environment**: part of VR dealing with surroundings
 - * **Virtual Artifacts**: part of VR dealing with simulated objects
 - * **Augmented Reality**: CG sensory overlay on real-world images
- **Collision Detection**
 - * **Static**: stationary objects (both not moving)
 - * **Dynamic**: moving objects (one or both)
 - * **Queries**
 - **Test-intersection**: determine whether objects do/will intersect
 - **Find-intersection**: calculate intersection set or contact set, time
 - * **Parametric methods**: use parameters to describe objects
 - * **Distance-based**: constrained minimization (closest points)
 - * **Intersection-based**: solving for parameters in equation

