Lecture 25 of 41

Spatial Sorting: Binary Space Partitioning Quadtrees & Octrees

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KSOL course pages: http://bit.ly/hGvXlH / http://bit.ly/eVizrE
Public mirror web site: http://www.kddresearch.org/Courses/CIS636
Instructor home page: http://www.cis.ksu.edu/~bhsu

Readings:
Today: Chapter 6, esp. §6.1, Eberly *2e* – see **http://bit.ly/ieUq45**
Next class: Chapter 7, §8.4, Eberly *2e*
Wikipedia, *Binary Space Partitioning*: **http://bit.ly/eE10lc**
Wikipedia, *Quadtree* (**http://bit.ly/ky0Xy**) & *Octree* (**http://bit.ly/dVrthx**)

Lecture Outline

- **Reading for Last Class: §2.4.3, 8.1, Eberly *2e*, GL handout**
- **Reading for Today: Chapter 6, Esp. §6.1, Eberly *2e***
- **Reading for Next Class: Chapter 7, §8.4, Eberly *2e***
- **Last Time: Collision Handling, Part 1 of 2**
  - **Static *vs.* dynamic objects, testing *vs.* finding intersections**
  - **Distance *vs.* intersection methods**
  - **Triangle point containment test**
  - **Method of separating axes**
- **Today: Adaptive Spatial Partitioning**
  - **Visible Surface Determination (VSD) revisited**
  - **Constructive Solid Geometry (CSG) trees**
  - **Binary Space Partitioning (BSP) trees**
  - **Quadtrees: adaptive 2-D (planar) subdivision**
  - **Octrees: adaptive 3-D (spatial) subdivision**
- **Coming Soon: Volume Graphics & Voxels**

Where We Are

21	**Lab 4a: Animation Basics**	**Flash animation handout**
22	Animation 2: Rotations; Dynamics, Kinematics	Chapter 17, esp. §17.1 – 17.2
23	Demos 4: Modeling & Simulation; Rotations	Chapter 10[1], 13[2], §17.3 – 17.5
24	**Collisions 1: axes, OBBs, Lab 4b**	**§2.4.3, 8.1, GL handout**
25	Spatial Sorting: Binary Space Partitioning	Chapter 6, esp. §6.1
26	**Demos 5: More CGA; Picking; HW/Exam**	**Chapter 7[2]; § 8.4**
27	**Lab 5a: Interaction Handling**	**§ 8.3 – 8.4; 4.2, 5.0, 5.6, 9.1**
28	Collisions 2: Dynamic, Particle Systems	§ 9.1, particle system handout
	Exam 2 review; Hour Exam 2 (evening)	**Chapters 5 – 6, 7[2] – 8, 12, 17**
29	**Lab 5b: Particle Systems**	**Particle system handout**
30	**Animation 3: Control & IK**	**§ 5.3, CGA handout**
31	Ray Tracing 1: intersections, ray trees	Chapter 14
32	Lab 6a: Ray Tracing Basics with POV-Ray	RT handout
33	Ray Tracing 2: advanced topic survey	Chapter 15, **RT handout**
34	Visualization 1: Data (Quantities & Evidence)	Tufte handout (1)
35	**Lab 6b: More Ray Tracing**	**RT handout**
36	Visualization 2: Objects	Tufte handout (2 & 4)
37	Color Basics; Term Project Prep	**Color handout**
38	Lab 7: Fractals & Terrain Generation	**Fractals/Terrain handout**
39	**Visualization 3: Processes; Final Review 1**	Tufte handout (3)
40	**Project presentations 1; Final Review 2**	**–**
41	Project presentations 2	–
	Final Exam	**Ch. 1 – 8, 10 – 15, 17, 20**

Lightly-shaded entries denote the due date of a written problem set; heavily-shaded entries, that of a machine problem (programming assignment); blue-shaded entries, that of a paper review; and the green-shaded entry, that of the term project.

Green, **blue** and **red** letters denote **exam review**, **exam**, and **exam solution review** dates.

Acknowledgements: Intersections, Containment – Eberly 1e

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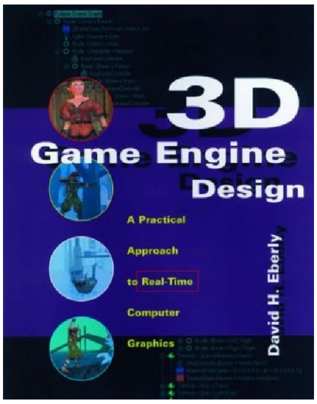

Last lecture's material:

- **View Frustum clipping**
  - §2.4.3, p. 70 – 77, 2e
  - §3.4.3, p. 93 – 99, & §3.7.2, p. 133 – 136, 1e
- **Collision detection: separating axes**
  - §8.1, p. 393 – 443, 2e
  - §6.4. p. 203 – 214, 1e

Later:

- **Distance methods**
  - Chapter 14, p. 639 – 679, 2e
  - §2.6, p. 38 – 77, 1e
- **Intersection methods**
  - Chapter 15, p. 681 – 717, 2e
  - §6.2 – 6.5, p. 188 – 243, 1e

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Review [1]: View Frustum Clipping

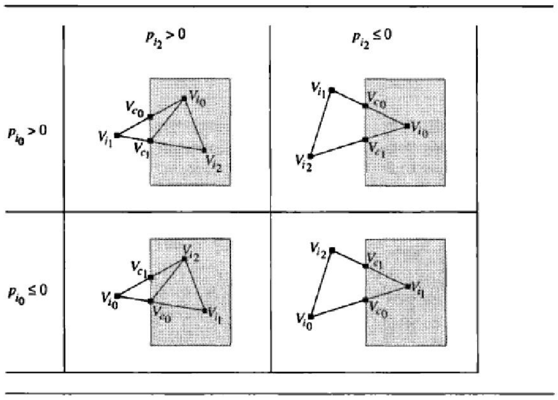

Figure 3.4 Four configurations for triangle splitting. Only the triangles in the shaded region are important, so the quadrilaterals outside are not split.

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Review [2]: Collision Detection vs. Response

- Collision Detection
  - Collision detection is a geometric problem
  - Given two moving objects defined in an initial and final configuration, determine if they intersected at some point between the two states
- Collision Response
  - The response to collisions is the actual physics problem of determining the unknown forces (or impulses) of the collision

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CSE169: Computer Animation, Winter 2005, http://bit.ly/f0ViAN

Review [3]: Queries – Test- vs. Find-Intersection

- Test-Intersection: Determine If Objects Intersect
  - Static: test whether they do at given instant
  - Dynamic: test whether they intersect at any point along trajectories
- Find-Intersection: Determine Intersection (or Contact) Set of Objects
  - Static: intersection set (compare: $A \cap B$)
  - Dynamic: contact time (interval of overlap), sets (depends on time)

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Review [4]: Queries – Distance vs. Intersection

- Distance-Based
  - Parametric representation of object boundaries/interiors
  - Want: closest points on two objects (to see whether they intersect)
  - Use: constrained minimization to solve for closest points
- Intersection-Based
  - Also uses parametric representation
  - Want: overlapping subset of interior of two objects
  - General approach: equate objects, solve for parameters
  - Use one of two kinds of solution methods
    - Analytical (when feasible to solve exactly – *e.g.*, OBBs)
    - Numerical (approximate region of overlap)
  - Solving for parameters in equation
  - Harder to compute than distance-based; use only when needed

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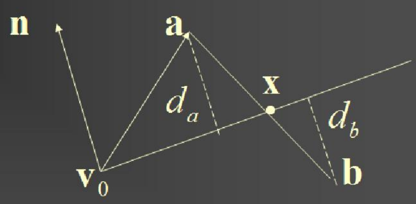
Review [5]: Segment vs. Triangle – Solution

- First, compute signed distances of a and b to plane

$$d_a = (\mathbf{a} - \mathbf{v}_0) \cdot \mathbf{n}$$
$$d_b = (\mathbf{b} - \mathbf{v}_0) \cdot \mathbf{n}$$



- Reject if both are above or both are below triangle
- Otherwise, find intersection point **x**

$$\mathbf{x} = \frac{d_a \mathbf{b} - d_b \mathbf{a}}{d_a - d_b}$$

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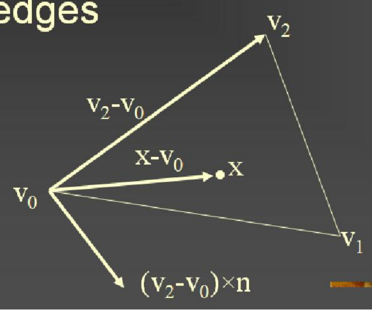
Review [6]: Segment vs. Triangle – Point Test

- Is point x inside the triangle?

$$(x\text{-}v_0) \cdot ((v_2\text{-}v_0) \times n) > 0$$

- Test all 3 edges



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Review [7]: Faster Triangle – Point Containment

- Reduce to 2D: remove smallest dimension
- Compute barycentric coordinates

$x' = x - v_0$

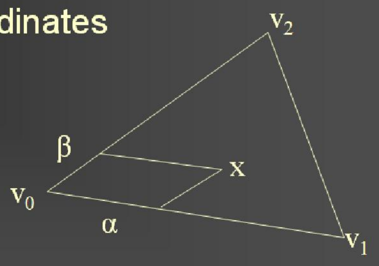
$e_1 = v_1 - v_0$

$e_2 = v_2 - v_0$

$\alpha = (x' \times e_2)/(e_1 \times e_2)$

$\beta = (x' \times e_1)/(e_1 \times e_2)$

- Reject if $\alpha<0$, $\beta<0$ or $\alpha+\beta>1$



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Review [8]: Sphere-Swept Volumes & Distances

Wikipedia: *Sphere*
http://bit.ly/9OWjQi
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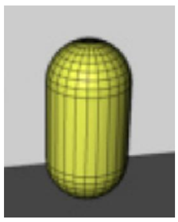

Capsule
Image © 2007 Remotion Wiki
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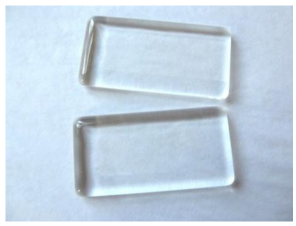

Lozenge
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Table 6.1 Relationship between sphere-swept volumes and distance calculators (*pnt*, point; *seg*, line segment; *rct*, rectangle).

	Sphere	*Capsule*	*Lozenge*
Sphere	dist(pnt,pnt)	dist(pnt,seg)	dist(pnt,rct)
Capsule	dist(seg,pnt)	dist(seg,seg)	dist(seg,rct)
Lozenge	dist(rct,pnt)	dist(rct,seg)	dist(rct,rct)

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Review [9]: Method of Separating Axes

Table 6.7 Values for R, R_0, and R_1 for the separating axis test $R > R_0 + R_1$ for two boxes in the direction of motion.

$\vec{L}$	R_0	R_1	R
$\vec{W} \times \vec{A}_0$	$a_1\|\alpha_2\| + a_2\|\alpha_1\|$	$\sum_{i=0}^{2} b_i\|c_{1i}\alpha_2 - c_{2i}\alpha_1\|$	$\|\vec{A}_0 \cdot \vec{W} \times \vec{D}\|$
$\vec{W} \times \vec{A}_1$	$a_0\|\alpha_2\| + a_2\|\alpha_0\|$	$\sum_{i=0}^{2} b_i\|c_{0i}\alpha_2 - c_{2i}\alpha_0\|$	$\|\vec{A}_1 \cdot \vec{W} \times \vec{D}\|$
$\vec{W} \times \vec{A}_2$	$a_0\|\alpha_1\| + a_1\|\alpha_0\|$	$\sum_{i=0}^{2} b_i\|c_{0i}\alpha_1 - c_{1i}\alpha_0\|$	$\|\vec{A}_2 \cdot \vec{W} \times \vec{D}\|$
$\vec{W} \times \vec{B}_0$	$\sum_{i=0}^{2} a_i\|c_{i1}\beta_2 - c_{i2}\beta_1\|$	$b_1\|\beta_2\| + b_2\|\beta_1\|$	$\|\vec{B}_0 \cdot \vec{W} \times \vec{D}\|$
$\vec{W} \times \vec{B}_1$	$\sum_{i=0}^{2} a_i\|c_{i0}\beta_2 - c_{i2}\beta_0\|$	$b_0\|\beta_2\| + b_2\|\beta_0\|$	$\|\vec{B}_1 \cdot \vec{W} \times \vec{D}\|$
$\vec{W} \times \vec{B}_2$	$\sum_{i=0}^{2} a_i\|c_{i0}\beta_1 - c_{i1}\beta_0\|$	$b_0\|\beta_1\| + b_1\|\beta_0\|$	$\|\vec{B}_2 \cdot \vec{W} \times \vec{D}\|$

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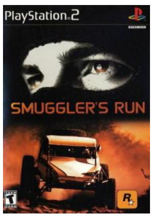
Acknowledgements: Collisions, BSP/Quadtrees/Octrees



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Data Structures for Scenes [1]: Four Tree Representations

- Scene Graphs
  - Organized by how scene is constructed
  - Nodes hold objects
- Constructive Solid Geometry (CSG) Trees
  - Organized by how scene is constructed
  - Leaves hold 3-D primitives
  - Internal nodes hold set operations
- Binary Space Partitioning (BSP) Trees
  - Organized by spatial relationships in scene
  - Nodes hold facets (in 3-D, polygons)
- Quadtrees & Octrees
  - Organized spatially
  - Nodes represent regions in space
  - Leaves hold objects

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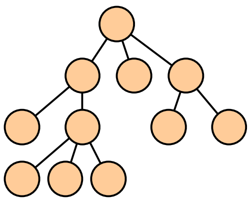
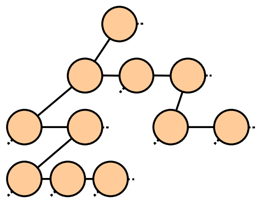
Data Structures for Scenes [2]: Implementing Scene Graphs

- We think of scene graphs as looking like the tree on the left.
- However, it is often convenient to implement them as shown on the right.
  - Implementation is a B-tree.
  - Child pointers are first-logical-child and next-logical-sibling.
  - Then traversing the logical tree is a simple pre-order traversal of the physical tree. This is how we draw.



Logical Tree



Physical Tree

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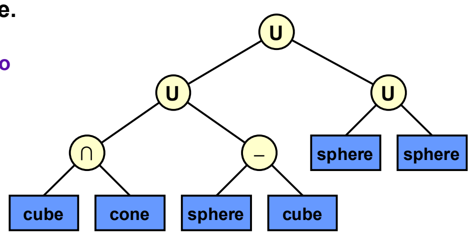
Data Structures for Scenes [3]: Constructive Solid Geometry Trees

- In Constructive Solid Geometry (CSG), we construct a scene out of primitives representing solid 3-D shapes. Existing objects are combined using set operations (union, intersection, set difference).
- We represent a scene as a binary tree.
  - Leaves hold primitives.
  - Internal nodes, which always have two children, hold set operations.
  - Order of children matters!



- CSG trees are useful for things other than rendering.
  - Intersection tests (collision detection, etc.) are not too hard. (Thus: ray tracing.)
- CSG does not integrate well with pipeline-based rendering, so we are not covering it in depth right now.
  - *How about a project on CSG?*

Binary Space Partitioning Trees [1]: Idea

- BSP tree: very different way to represent a scene
  - Nodes hold facets
  - Structure of tree encodes spatial information about the scene
- Applications
  - Visible Surface Determination (VSD) *aka* Hidden Surface Removal
  - Wikipedia: *Visible Surface Determination*, http://bit.ly/et2yNQ
  - Related applications: portal rendering (http://bit.ly/fYO5T6), *etc.*

Binary Space Partitioning Trees [2]: Definition

- BSP tree: type of binary tree
  - Nodes can have 0, 1, or two children
  - Order of child nodes matters, and if a node has just 1 child, it matters whether this is its left or right child
- Each node holds a facet
  - This may be only part of a facet from original scene
  - When constructing a BSP tree, we may need to split facets
- Organization
  - Each facet lies in a unique plane
    - ⇨ In 2-D, a unique line
  - For each facet, we choose one side of its plane to be "outside" Other direction: "inside"
    - ⇨ This can be the side the normal vector points toward
  - Rule: For each node
    - ⇨ Its left descendant subtree holds only facets "inside" it
    - ⇨ Its right descendant subtree holds only facets "outside" it

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Binary Space Partitioning Trees [3]: Construction

- To construct a BSP tree, we need
  - List of facets (with vertices)
  - "Outside" direction for each
- Procedure
  - Begin with empty tree
  - Iterate through facets, adding new node to tree for each new facet
  - First facet goes in root node.
  - For each subsequent facet, descend through tree, going left or right depending on whether facet lies inside or outside the facet stored in relevant node
    - ⇨ If facet lies partially inside & partially outside, split it along plane [line] of facet
    - ⇨ Facet becomes two "partial" facets
    - ⇨ Each inherits its "outside" direction from original facet
    - ⇨ Continue descending through tree with each partial facet separately
  - Finally, (partial) facet is added to current tree as leaf

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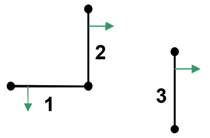
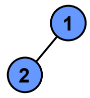
Binary Space Partitioning Trees [4]: Simple Example

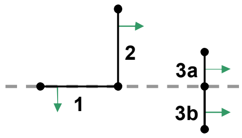
- Suppose we are given the following (2-D) facets and "outside" directions:



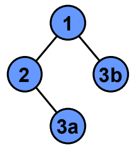
- We iterate through the facets in numerical order
  - Facet 1 becomes the root
  - Facet 2 is inside of 1
  - Thus, after facet 2, we have the following BSP tree:



- Facet 3 is partially inside facet 1 and partially outside.
  - We split facet 3 along the line containing facet 1
  - The resulting facets are 3a and 3b
  - They inherit their "outside" directions from facet 3

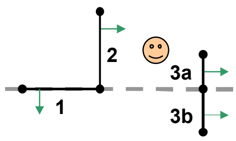


- We place facets 3a and 3b separately
  - Facet 3a is inside facet 1 and outside facet 2
  - Facet 3b is outside facet 1
- The final BSP tree looks like this:



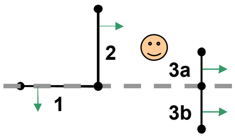
BSP Tree Traversal [1]

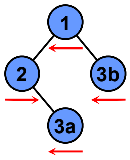
- **Important use of BSP trees: provide back-to-front (or front-to-back) ordering of facets in scene, from point of view of observer**
  - **When we say "back-to-front" ordering, we mean that no facet comes before something that appears directly behind it**
  - **This still allows nearby facets to precede those farther away**
  - **Key idea: All descendants on one side of facet can come before facet, which can come before all descendants on other side**
- **Procedure**
  - **For each facet, determine on which side of it observer lies**
  - **Back-to-front ordering: in-order traversal of tree where subtree opposite from observer comes before subtree on same side**



BSP Tree Traversal [2]

- Procedure:
  - For **each facet**, determine on which side of it the observer lies.
  - Back-to-front ordering: Do an in-order traversal of the tree in which the subtree opposite from the observer comes before the subtree on the same side as the observer.
- Our observer is inside 1, outside 2, inside 3a, outside 3b.





- Resulting back-to-front ordering: 3b, 1, 2, 3a.
- Is this really back-to-front?

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BSP Trees: What Are They Good For?

- **BSP trees are primarily useful when a back-to-front or front-to-back ordering is desired:**
  - **For HSR**
  - **For translucency via blending**
- **Since it can take some time to construct a BSP tree, they are useful primarily for:**
  - **Static scenes**
  - **Some dynamic objects are acceptable**
- **BSP-tree techniques are generally a waste of effort for small scenes. We use them on:**
  - **Large, complex scenes**

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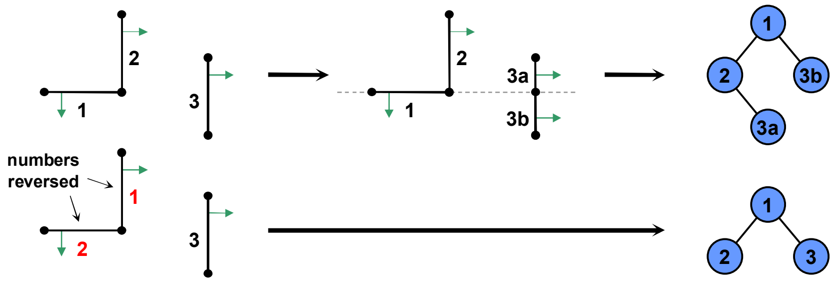
BSP Tree Optimization

- Order in which we iterate through the facets can matter a great deal
  - Consider our simple example again
  - If we change the ordering, we can obtain a simpler BSP tree



- If a scene is not going to change, and the BSP tree will be used many times, then it may be worth a large amount of preprocessing time to find the best possible BSP tree

BSP Trees: Finding Inside/Outside [1]

- When dealing with BSP trees, we need to determine inside or outside many times. What exactly does this mean?
  - A facet lies entirely on one side of a plane if all of its vertices lie on that side.
  - Vertices are points. The position of the observer is also a point.
  - Thus, given a facet and a point, we need to be able to determine on which side of the facet's plane the point lies.
- We assume we know the normal vector of the facet (and that it points toward the "outside").
  - If not, compute the normal using a cross product.
  - If you are using `vecpos.h`, and three non-colinear vertices of the facet are stored in `pos` variables `p1`, `p2`, `p3`, then you can find the normal as follows.

```
vec n = cross(p2-p1, p3-p1).normalized();
```

BSP Trees: Finding Inside/Outside [2]

- To determine on which side of a facet's plane a point lies:
  - Let N be the normal vector of the facet
  - Let p be a point in the facet's plane
    - Maybe p is a vertex of the facet?
  - Let z be the point we want to check
  - Compute (z – p) · N
    - If this is positive, then z is on the outside
    - Negative: inside
    - Zero: on the plane
- Using `vecpos.h`, and continuing from previous slide:

```
pos z = …;  // point to check
if (dot(z-p1, n) >= 0.)
   // Outside or on plane
else
   // Inside
```

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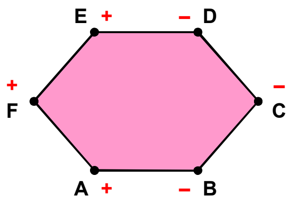
BSP Trees: Splitting Polygons [1]

- May need to split facet when constructing BSP tree
- Example
  - Suppose we have the facet shown below.
  - If all vertices are (say) outside, then no split required
  - But if A, E, and F are outside (+), and B, C, and D are inside (–), then we must split into two facets



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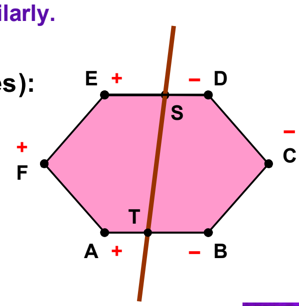
BSP Trees: Splitting Polygons [2]

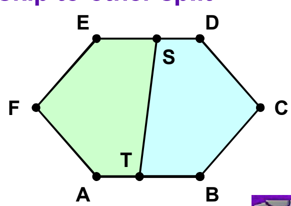
- Where do we split?
  - Since the expression (z – p) · N is positive at E and negative at D, it must be zero somewhere on the line segment joining D and E. Call this point S. This is one place where the facet splits.
  - Let k_1 be the value of (z – p) · N at D, and let k_2 be the value at E.
  - Then S = $(1/(k_2 – k_1)) (k_2D – k_1E)$.
  - Point T (shown in the diagram) is computed similarly.
- Using `vecpos.h` (continuing from earlier slides):

```
double k1 = dot(D-p1, n);
double k2 = dot(E-p1, n);
pos S = affinecomb(k2, D, -k1, E);
// Explanation of above line?
```



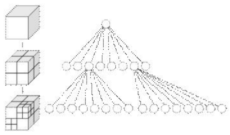
BSP Trees: Splitting Polygons [3]

- We were given vertices A, B, C, D, E, F in order
- We computed S and T
  - S lies between D and E
  - T lies between A and B
- We have A, (split at T), B, C, D, (split at S), E, F
- We form two polygons as follows:
  - Start through vertex list
  - When we get to split, use that vertex, and skip to other split
  - Result: A, T, S, E, F
  - Do the same with the part we skipped
  - Result: B, C, D, S, T



Quadtrees & Octrees [1]: Background

- Idea of binary space partition: good general applicability
- Variations used in several different structures
  - BSP trees (of course)
    - ⇨ Split along planes containing facets
  - Quadtrees & octrees (next)
    - ⇨ Split along pre-defined planes.
  - *K-d* trees (Lecture 28)
    - ⇨ Split along planes parallel to coordinate axes, so as to split up the objects nicely.
    - ⇨ *How about a project on K-d trees?*
- Quadtrees used to partition 2-D space; octrees are for 3-D
  - Two concepts are nearly identical
  - Unfortunate that they are given different names



Wikipedia, *Octree*
http://bit.ly/dVrthx

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Quadtrees & Octrees [2]: Definition

- In general
  - Quadtree: tree in which each node has at most 4 children
  - Octree: tree in which each node has at most 8 children
  - Binary tree: tree in which each node has at most 2 children
- In practice, however, we use "quadtree" and "octree" to mean something more specific
  - Each node of the tree corresponds to a square (quadtree) or cubical (octree) region
  - If a node has children, think of its region being chopped into 4 (quadtree) or 8 (octree) equal subregions
  - Child nodes correspond to these smaller subregions of parent's region
  - Subdivide as little or as much as is necessary
  - Each internal node has exactly 4 (quadtree) or 8 (octree) children

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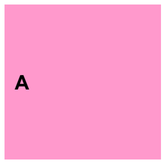
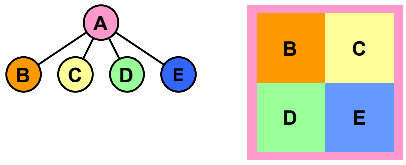
Quadtrees & Octrees [3]: Example

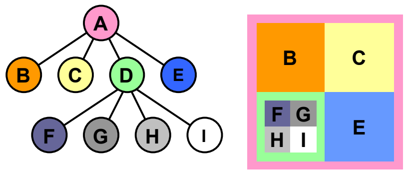
- Root node of quadtree corresponds to square region in space
  - Generally, this encompasses entire “region of interest”



- If desired, subdivide along lines parallel to the coordinate axes, forming four smaller identically sized square regions
  - Child nodes correspond to these



- Some or all of these children may be subdivided further
- Octrees work in a similar fashion, but in 3-D, with cubical regions subdivided into 8 parts



Quadtrees & Octrees [4]: What Are They Good For?

- Handling Observer-Object Interactions
  - Subdivide the quadtree/octree until each leaf’s region intersects only a small number of objects
  - Each leaf holds a list of pointers to objects that intersect its region
  - Find out which leaf the observer is in. We only need to test for interactions with the objects pointed to by that leaf
- Inside/Outside Tests for Odd Shapes
  - The root node represent a square containing the shape
  - If node’s region lies entirely inside or entirely outside shape, do not subdivide it
  - Otherwise, do subdivide (unless a predefined depth limit has been exceeded)
  - Then the quadtree or octree contains information allowing us to check quickly whether a given point is inside the shape
- Sparse Arrays of Spatially-Organized Data
  - Store array data in the quadtree or octree
  - Only subdivide if that region of space contains interesting data
  - This is how an octree is used in the BLUIsculpt program

Summary

- Reading for Last Class: §2.4.3, 8.1, Eberly *2e*, GL handout
- Reading for Today: Chapter 6, Esp. §6.1, Eberly *2e*
- Reading for Next Class: Chapter 7, §8.4, Eberly *2e*
- Last Time: Collision Detection Part 1 of 2
  - Static *vs.* dynamic, testing *vs.* finding, distance *vs.* intersection
  - Triangle point containment test
  - Lots of intersections: spheres, capsules, lozenges
  - Method of separating axes
- Today: Adaptive Spatial Partitioning
  - Visible Surface Determination (VSD) revisited
  - Constructive Solid Geometry (CSG) trees
  - Binary Space Partitioning (BSP) trees
  - Quadtrees: adaptive 2-D (planar) subdivision
  - Octrees: adaptive 3-D (spatial) subdivision
- Coming Soon: Volume Graphics & Voxels

Terminology

- Collision Detection
  - Static vs. dynamic objects
  - Queries: test-intersection *vs.* find-intersection
  - Parametric methods: distance-based, intersection-based
- Bounding Objects
  - Axis-aligned bounding box
  - Oriented bounding box: can point in arbitrary direction
  - Sphere
  - Capsule
  - Lozenge
- Constructive Solid Geometry Tree: Regularized Boolean Set Operators
- Adaptive Spatial Partitioning: Calculating Intersection, Visibility
  - Binary Space Partitioning tree – 2-way decision tree/surface
  - Quadtree – 4-way for 2-D
  - Octree – 8-way for 3-D