

## Lecture 30 of 41

# Animation 3 of 3: Inverse Kinematics Control & Ragdoll Physics

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KSOL course pages: <http://bit.ly/hGvXIH> / <http://bit.ly/eVizrE>

Public mirror web site: <http://www.kddresearch.org/Courses/CIS636>

Instructor home page: <http://www.cis.ksu.edu/~bhsu>

### Readings:

Last class: **Particle System Handout**

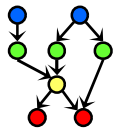
Today: §5.3, Eberly 2<sup>e</sup> – see <http://bit.ly/ieUq45>; **CGA Handout**

Next class: Chapter 14, Eberly 2<sup>e</sup>

Reference: Wikipedia, *Inverse Kinematics*, <http://bit.ly/hr8r2u>

Reference: Wikipedia, *Ragdoll Physics*, <http://bit.ly/3oggUZ>

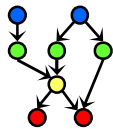




## Lecture Outline

- Reading for Last Class: **Particle System Handout**
- Reading for Today: §5.3, Eberly 2<sup>e</sup>; **CGA Handout**
- Reading for Next Class: Chapter 14, Eberly 2<sup>e</sup>
- Last Time: Lab on Particle Systems; Dissection of Working Program
- Today: Animation Part 3 of 3 – Inverse Kinematics
  - \* Autonomous agents (robots, swarms) vs. hand-animated movement
  - \* Forward kinematics and control
  - \* Inverse kinematics for autonomous movement in robotics
  - \* Jacobians and iterative minimization models
  - \* Rag doll physics
- End of Material on: Particle Systems, Collisions, CGA
- Also Conclusion of Physically-Based Modeling (PBM)
- Next Class: Ray Tracing, Part 1 of 2
  - \* **V**ectors: light/shadow (L), reflected (R), transmitted/refracted (T)
  - \* Basic recursive ray tracing: ray trees





## Where We Are

|    |   |  |
|----|---|--|
| 21 | Lab 4a: Animation Basics                      | Flash animation handout                                  |
| 22 | Animation 2: Rotations; Dynamics, Kinematics  | Chapter 17, esp. §17.1 – 17.2                            |
| 23 | Demos 4: Modeling & Simulation; Rotations     | Chapter 10 <sup>1</sup> , 13 <sup>2</sup> , §17.3 – 17.5 |
| 24 | Collisions 1: axes, OBBs, Lab 4b              | §2.4.3, 8.1, GL handout                                  |
| 25 | Spatial Sorting: Binary Space Partitioning    | Chapter 6, esp. §6.1                                     |
| 26 | Demos 5: More CGA; Picking; HW/Exam           | Chapter 7 <sup>2</sup> ; § 8.4                           |
| 27 | Lab 5a: Interaction Handling                  | § 8.3 – 8.4; 4.2, 5.0, 5.6, 9.1                          |
| 28 | Collisions 2: Dynamic, Particle Systems       | § 9.1, particle system handout                           |
|    | Exam 2 review; Hour Exam 2 (evening)          | Chapters 5 – 6, 7 <sup>2</sup> – 8, 12, 17               |
| 29 | Lab 5b: Particle Systems                      | Particle system handout                                  |
| 30 | Animation 3: Control & IK                     | § 5.3, CGA handout                                       |
| 31 | Ray Tracing 1: intersections, ray trees       | Chapter 14   |
| 32 | Lab 6a: Ray Tracing Basics with POV-Ray       | RT handout   |
| 33 | Ray Tracing 2: advanced topic survey          | Chapter 15, RT handout                                   |
| 34 | Visualization 1: Data (Quantities & Evidence) | Tufte handout (1)  |
| 35 | Lab 6b: More Ray Tracing                      | RT handout   |
| 36 | Visualization 2: Objects                      | Tufte handout (2 & 4)                                    |
| 37 | Color Basics; Term Project Prep               | Color handout  |
| 38 | Lab 7: Fractals & Terrain Generation          | Fractals/Terrain handout                                 |
| 39 | Visualization 3: Processes; Final Review 1    | Tufte handout (3)  |
| 40 | Project presentations 1; Final Review 2       | –  |
| 41 | Project presentations 2                       | –  |
|    | Final Exam                                    | Ch. 1 – 8, 10 – 15, 17, 20                               |

Lightly-shaded entries denote the due date of a written problem set; heavily-shaded entries, that of a machine problem (programming assignment); blue-shaded entries, that of a paper review; and the green-shaded entry, that of the term project.

Green, blue and red letters denote exam review, exam, and exam solution review dates.





## Acknowledgements: Inverse Kinematics



### David C. Brogan

Visiting Assistant Professor, Computer Science Department, University of Virginia

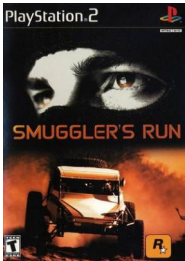
<http://www.cs.virginia.edu/~dbrogan/>

Susquehanna International Group (SIG)

<http://www.sig.com>



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<http://micromachine.stanford.edu/~rmelamud/>





## Review [1]: Uses of Particle Systems

- **Explosions**
  - ✱ Large
  - ✱ Fireworks
- **Fire**
- **Vapor**
  - ✱ Clouds
  - ✱ Dust
  - ✱ Fog
  - ✱ Smoke
  - ✱ Contrails
- **Water**
  - ✱ Waterfalls
  - ✱ Streams
- **Plants**



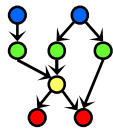
Command & Conquer 4: Tiberian Twilight

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Wikipedia: <http://bit.ly/gFGMjO>

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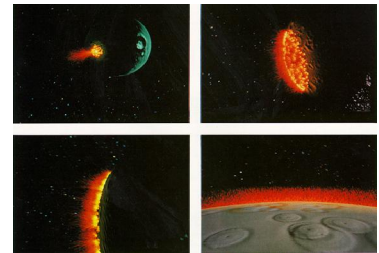
## Review [2]: History of Particle Systems



*Spacewar!* © 1962 S. Russell et al.  
Wikipedia: <http://bit.ly/eaiWUW>



*Asteroids* © 1979 L. Rains & E. Logg  
Wikipedia: <http://bit.ly/hwfEQk>

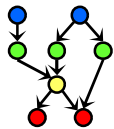


*Star Trek II* © 1983 Paramount  
Wikipedia: <http://bit.ly/eXwrhb>

- *Spacewar!* (1962) Used Pixel Clouds as Explosions
- *Asteroids* (1979) First “Physically-Based” PS/Collision Model in Games
- *Star Trek II* (1983) Particle Fountain: <http://youtu.be/Qe9qSLYK5q4>
- *Hey, Hey, 16K* © 2000 M. J. Hibbett, Video © 2004 R. Manuel  
<http://youtu.be/Ts96J7HhO28>

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## Review [3]: Definition & Physically-Based Model

- A particle system is a collection of a number of individual elements or *particles*.
- *Particle systems control a set of particles that act autonomously but share some common attributes.*
- Particle is a point in 3D space.
- Forces (e.g. gravity or wind) accelerate a particle.
- Acceleration changes velocity.
- Velocity changes position

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## Review [4]: More Attributes of Particles

- Position
- Velocity
- Life Span
- Size
- Weight
- Representation
- Color
- Owner

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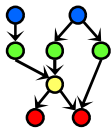


## Review [5]: Four Ways to Represent Particles

- Points
- Lines
- Texture-mapped quads
- Point Sprites

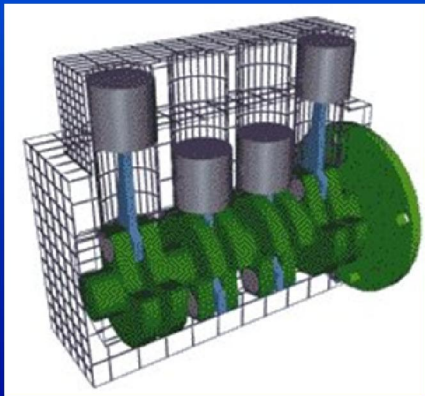
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## Kinematics

- The study of object movements irrespective of their speed or style of movement



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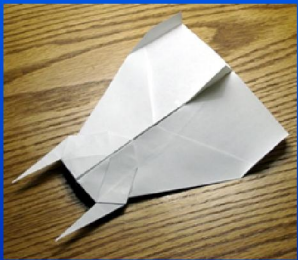


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## Degrees of Freedom (DOFs) [1]: Translational & Rotational

- The variables that affect an object's orientation
  - How many degrees of freedom when flying?
- 
- So the kinematics of this airplane permit movement anywhere in three dimensions
  - Six
    - x, y, and z positions
    - roll, pitch, and yaw

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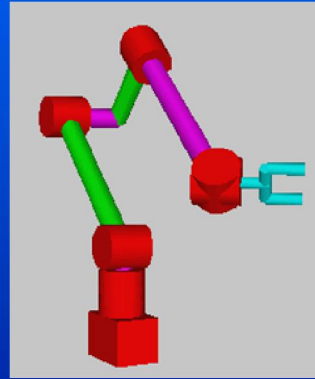
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## Degrees of Freedom (DOFs) [2]: Robot Arm

- How about this robot arm?



- Six again
  - 2-base, 1-shoulder, 1-elbow, 2-wrist

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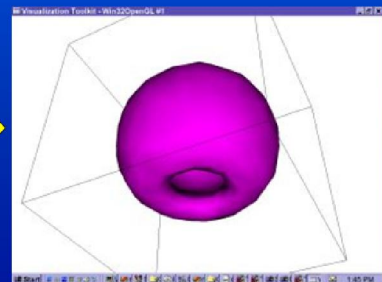
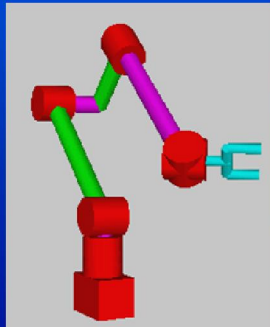
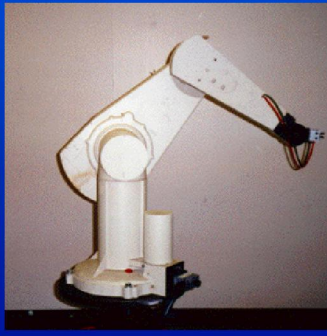
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## Configuration Space

- The set of all possible positions (defined by kinematics) an object can attain



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## Work Space vs. Configuration Space

- Work space
  - The space in which the object exists
  - Dimensionality
    - $R^3$  for most things,  $R^2$  for planar arms
- Configuration space
  - The space that defines the possible object configurations
  - Degrees of Freedom
    - The number of parameters that necessary and sufficient to define position in configuration

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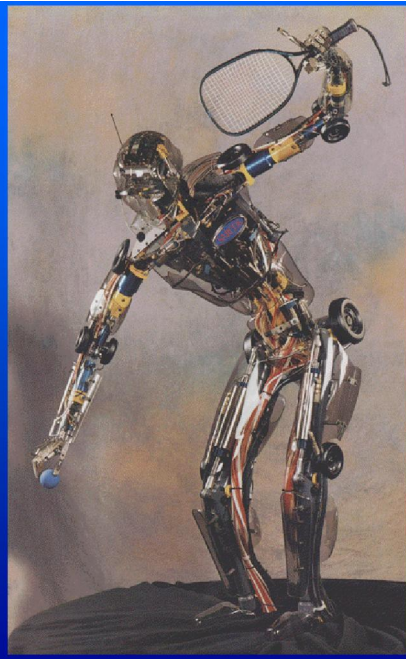
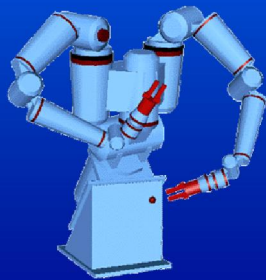
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## More Examples

- A point on a plane
- A point in space
- A point moving on a line in space



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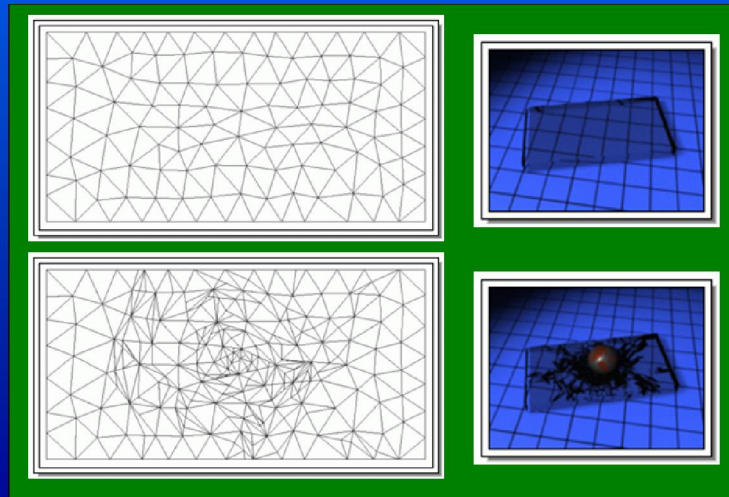






## Controlled DOFs

- DOFs that you can actually control (position explicitly)



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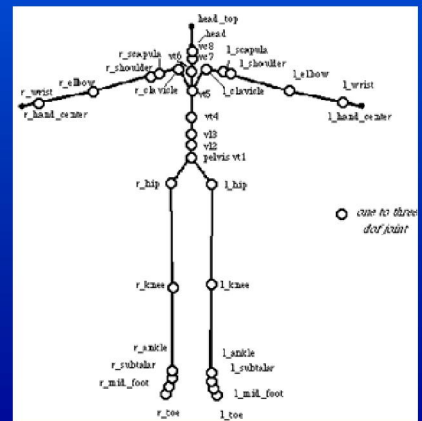
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## Hierarchical Kinetic Modeling

- A family of parent-child spatial relationships are functionally defined
  - Moon/Earth/Sun movements
  - Articulations of a humanoid
- Limb connectivity is built into model (joints) and animation is easier



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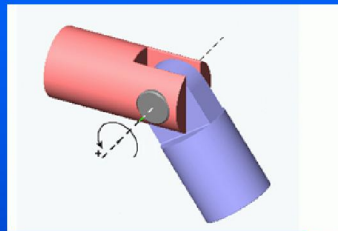
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## Robot Parts & Terms

- Links
- End effector
- Frame
- Revolute Joint
- Prismatic Joint



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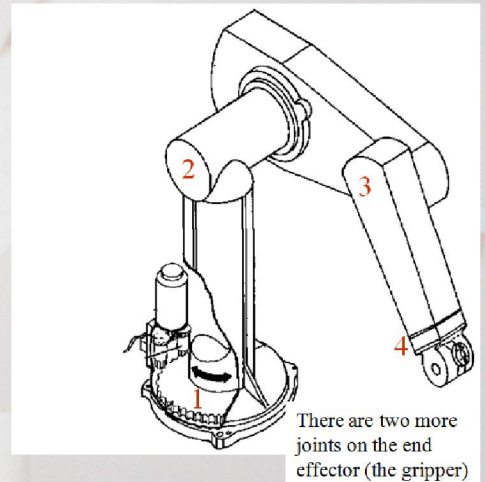
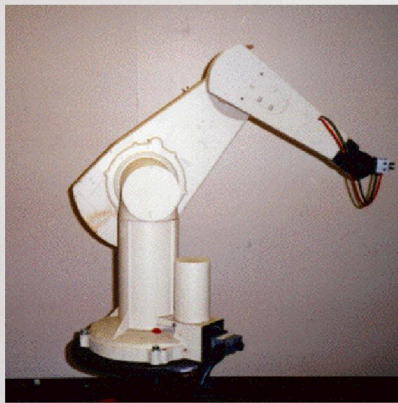


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## Example: Puma 560 Robot

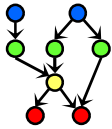


The PUMA 560 has **SIX** revolute joints  
A revolute joint has ONE degree of freedom ( 1 DOF) that is defined by its angle

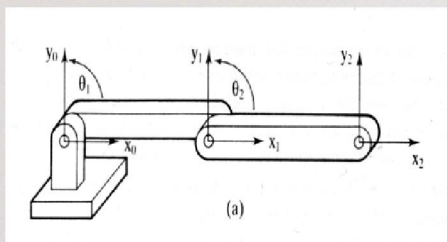
Wikipedia, *Programmable Universal Machine for Assembly (PUMA)*: <http://bit.ly/fBMRaM>

Adapted from slides ♥ 2002 R. Melamud, Stanford University  
Mirrored at CMU 16-311 Introduction to Robotics, <http://generalrobotics.org>

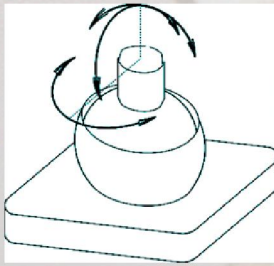




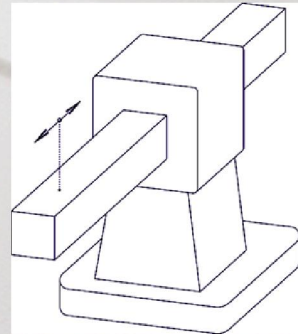
## Joint Types: Revolute, Prismatic, Spherical



Revolute Joint  
1 DOF ( Variable -  $\theta$  )



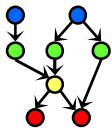
Prismatic Joint  
1 DOF (linear) (Variables -  $d$ )



Spherical Joint  
3 DOF ( Variables -  $\theta_1, \theta_2, \theta_3$  )

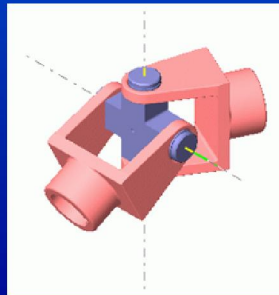
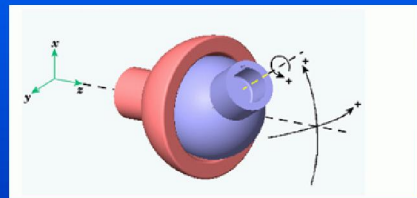
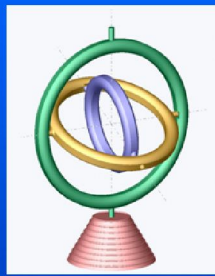
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Mirrored at CMU 16-311 Introduction to Robotics, <http://generalrobotics.org>





## More Complex Joints

- 3 DOF joints
  - Gimbal
  - Spherical (doesn't possess singularity)
- 2 DOF joints
  - Universal



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## Hierarchical Representation

- Model bodies (links) as nodes of a tree
- All body frames are local (relative to parent)
  - Transformations affecting root affect all children
  - Transformations affecting any node affect all its children

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## Forward vs. Inverse Kinematics

- Forward Kinematics
  - Compute configuration (pose) given individual DOF values
- Inverse Kinematics
  - Compute individual DOF values that result in specified end effector position

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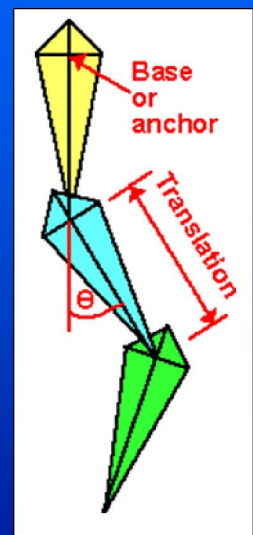
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## Forward Kinematics [1]: Definition & General Approach

- Traverse kinematic tree and propagate transformations downward
  - Use stack
  - Compose parent transformation with child's
  - Pop stack when leaf is reached
- High DOF models are tedious to control this way



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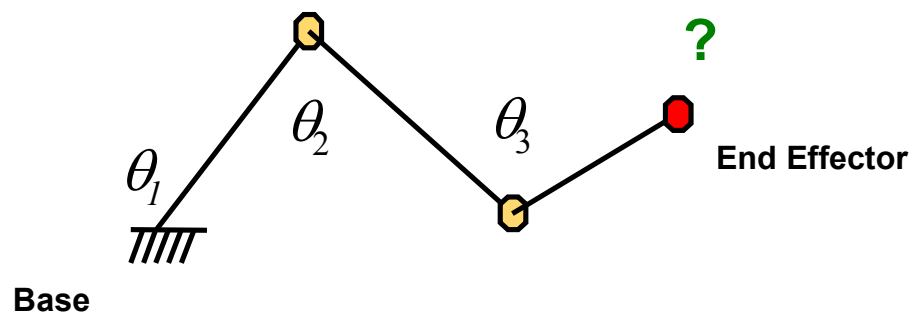


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## Forward Kinematics [2]: Illustration



$$\vec{x} = f(\vec{\theta})$$

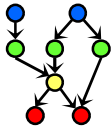
Choi

$$\mathbf{e} = f(\Phi)$$

Rotenberg

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Graphics and Media Lab (<http://graphics.snu.ac.kr>) – mirrored at: <http://bit.ly/hnzSAN>





## Forward Kinematics [3]: Joint Angles to Bone Coordinates

- The local and world matrix construction within the skeleton is an implementation of *forward kinematics*
- Forward kinematics refers to the process of computing world space geometric descriptions (matrices...) based on joint DOF values (usually rotation angles and/or translations)

Adapted from slides ♥ 2004 – 2005 S. Rotenberg, UCSD  
CSE169: Computer Animation, Winter 2005 – <http://bit.ly/f0ViAN>





## Inverse Kinematics [1]: Definition & General Approach

- Given end effector position, compute required joint angles
- In simple case, analytic solution exists
  - Use trig, geometry, and algebra to solve

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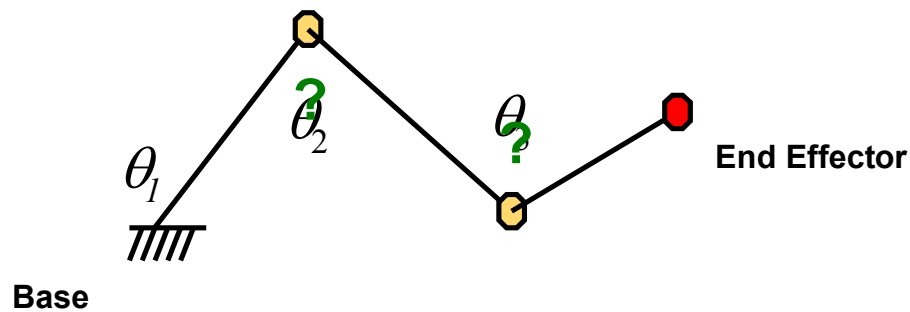
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## Inverse Kinematics [2]: Illustration

For more on characters & IK, see:  
Advanced Topics in CG Lecture 05



$$\vec{\theta} = f^{-1}(\vec{x}) \quad \Phi = f^{-1}(\mathbf{e})$$

Choi                      Rotenberg

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## Inverse Kinematics [3]: Demos

Inverse Kinematics demo



© 2008 M. Kinzelman  
<http://youtu.be/I52yZ491kPo>

Inverse Kinematics Demonstration in Maya



© 2007 A. Brown  
<http://youtu.be/6JdLOLazJJ0>

Momentum-based Inverse Kinematics with Motion Capture



© 2008 T. Komura, H. S. Lim, & R. W. H. Lau  
<http://youtu.be/FJTBmNP6oCM>

PUMA robot playing golf



© 2011 K. Iyer  
<http://youtu.be/YvRBWIRAPsE>







## Inverse Kinematics [4]: Analytic Solution for 2-Link Case

$$x^2 + y^2 = a_1^2 + a_2^2 - 2a_1a_2 \cos(\pi - \theta_2)$$

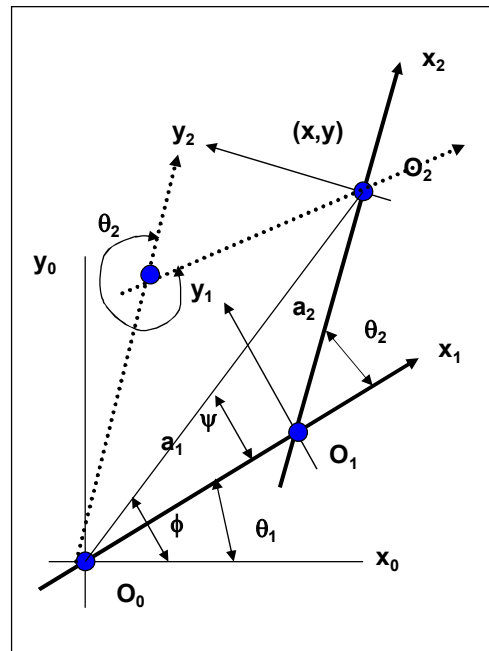
$$\cos \theta_2 = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

for greater accuracy

$$\begin{aligned} \tan^2 \frac{\theta_2}{2} &= \frac{1 - \cos \theta}{1 + \cos \theta} = \frac{2a_1a_2 - x^2 - y^2 + a_1^2 + a_2^2}{2a_1a_2 + x^2 + y^2 - a_1^2 - a_2^2} \\ &= \frac{(a_1^2 + a_2^2)^2 - (x^2 + y^2)}{(x^2 + y^2) - (a_1^2 - a_2^2)^2} \end{aligned}$$

$$\theta_2 = \pm 2 \tan^{-1} \sqrt{\frac{(a_1^2 + a_2^2)^2 - (x^2 + y^2)}{(x^2 + y^2) - (a_1^2 - a_2^2)^2}}$$

Two solutions: elbow up & elbow down



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## Inverse Kinematics [5]: Iterative IK Solutions

- Frequently analytic solution is infeasible
- Use **Jacobian**
- Derivative of function output relative to each of its inputs
- If  $y$  is function of three inputs and one output

$$y = f(x_1, x_2, x_3)$$

$$\delta y = \frac{\delta f}{\partial x_1} \cdot \delta x_1 + \frac{\delta f}{\partial x_2} \cdot \delta x_2 + \frac{\delta f}{\partial x_3} \cdot \delta x_3$$

- Represent Jacobian  $J(X)$  as a 1x3 matrix of partial derivatives

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## Jacobian [1]: 6x6 DOF Case

- In another situation, end effector has 6 DOFs and robotic arm has 6 DOFs
- $f(x_1, \dots, x_6) = (x, y, z, r, p, y)$
- Therefore  $J(X) = 6 \times 6$  matrix

$$\begin{bmatrix} \frac{\partial f_x}{\partial x_1} & \frac{\partial f_y}{\partial x_1} & \frac{\partial f_z}{\partial x_1} & \frac{\partial f_r}{\partial x_1} & \frac{\partial f_p}{\partial x_1} & \frac{\partial f_y}{\partial x_1} \\ \frac{\partial f_x}{\partial x_2} & \frac{\partial f_y}{\partial x_2} & \frac{\partial f_z}{\partial x_2} & \frac{\partial f_r}{\partial x_2} & \frac{\partial f_p}{\partial x_2} & \frac{\partial f_y}{\partial x_2} \\ \frac{\partial f_x}{\partial x_3} & \frac{\partial f_y}{\partial x_3} & \frac{\partial f_z}{\partial x_3} & \frac{\partial f_r}{\partial x_3} & \frac{\partial f_p}{\partial x_3} & \frac{\partial f_y}{\partial x_3} \\ \frac{\partial f_x}{\partial x_4} & \frac{\partial f_y}{\partial x_4} & \frac{\partial f_z}{\partial x_4} & \frac{\partial f_r}{\partial x_4} & \frac{\partial f_p}{\partial x_4} & \frac{\partial f_y}{\partial x_4} \\ \frac{\partial f_x}{\partial x_5} & \frac{\partial f_y}{\partial x_5} & \frac{\partial f_z}{\partial x_5} & \frac{\partial f_r}{\partial x_5} & \frac{\partial f_p}{\partial x_5} & \frac{\partial f_y}{\partial x_5} \\ \frac{\partial f_x}{\partial x_6} & \frac{\partial f_y}{\partial x_6} & \frac{\partial f_z}{\partial x_6} & \frac{\partial f_r}{\partial x_6} & \frac{\partial f_p}{\partial x_6} & \frac{\partial f_y}{\partial x_6} \end{bmatrix}$$

Adapted from slides ♥ 2000 – 2005 D. Brogan, University of Virginia  
CS 551, Advanced CG & Animation – <http://bit.ly/hUXrqd>



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## Jacobian [2]: Solution

- Relates velocities in parameter space to velocities of outputs

$$\dot{Y} = J(X) \cdot \dot{X}$$

- If we know  $Y_{\text{current}}$  and  $Y_{\text{desired}}$ , then we subtract to compute  $\dot{Y}_{\text{dot}}$
- Invert Jacobian and solve for  $\dot{X}_{\text{dot}}$

Adapted from slides ♥ 2000 – 2005 D. Brogan, University of Virginia  
CS 551, Advanced CG & Animation – <http://bit.ly/hUXrqd>

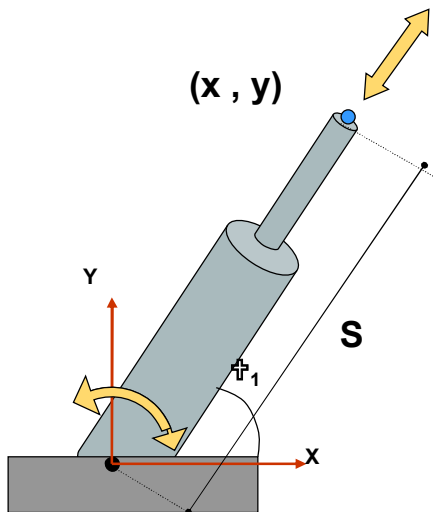


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## Another IK Problem: Revolute & Prismatic Joints Combined



Finding  $\theta$ :

$$\theta = \arctan\left(\frac{y}{x}\right)$$

More Specifically:

$$\theta = \arctan 2\left(\frac{y}{x}\right)$$

**arctan2()** specifies  
that it's in the first  
quadrant

Finding  $S$ :

$$S = \sqrt{x^2 + y^2}$$

Adapted from slides ♥ 2002 R. Melamud, Stanford University  
Mirrored at CMU 16-311 Introduction to Robotics, <http://generalrobotics.org>





## Ragdoll Physics [1]: Definition

- **Type of Procedural Animation**

- ✦ Automatically generates CGA directives (rotations)
- ✦ Based on simulation
- ✦ Rigid-body dynamics

- **Articulated Figure**

- ✦ Gravity
- ✦ No autonomous movement
- ✦ Used for inert body
  - Usually: character death (car impact, falling body, etc.)
  - Less often: unconscious, paralyzed character

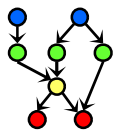


*Falling Bodies* © 1997 – 2001 Animats  
<http://www.animats.com>

- **Collisions with Multiple Bodies**

- ✦ Inter-character
- ✦ Character-object





## Ragdoll Physics [2]: Demos

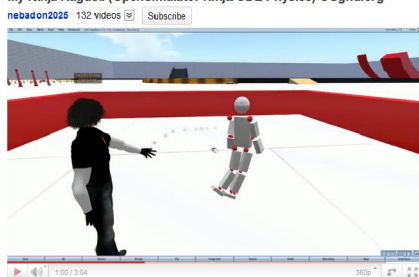
3ds max 8 Ragdoll Physics Test



© 2007 N. Picouet

<http://youtu.be/ohNqCb--aSs>

My Ninja Ragdoll (OpenSimulator Ninja/ODE Physics) OSgrid.org



© 2009 M. E. Cerquoni

[http://youtu.be/uW\\_DK2qvKv8](http://youtu.be/uW_DK2qvKv8)

Ragdoll physics



© 2006 P. Pelt

<http://youtu.be/6JdLOLazJJ0>

See also: [http://youtu.be/5\\_Qlsl0fyaU](http://youtu.be/5_Qlsl0fyaU)

Ragdoll Demo (Python + ODE)



© 2010 M. Heinzen (Arkain)

<http://bit.ly/gUj9Su> / <http://youtu.be/FJTBMnP6oCM>







## Physically-Based Modeling (PBM) [1]: Looking Back

- **Particle Dynamics**

- ★ **Emitters**

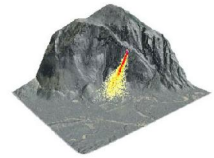
- 0-D (points), 1-D (lines), 2-D (planes, discs, cross-sections)
    - e.g., fireworks (0-D); fountains (0/1/2-D); smokestacks, jets (2-D)

- ★ **Simulation: birth-death process, functions of particle age/trajectory**

- **Rigid-Body Dynamics**

- ★ **Constrained systems of connected parts**

- ★ **Examples: falling rocks, colliding vehicles, rag dolls**



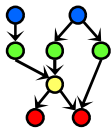
Rocks fall  
Everyone dies

- **Articulated Figures**

- **More References**

- ★ **ACM, *Intro to Physically-Based Modeling*: <http://bit.ly/hhQvXd>**
  - ★ **Wikipedia, *Physics Engine*: <http://bit.ly/h4PIRt>**
  - ★ **Wikipedia, *N-Body Problem*: <http://bit.ly/1ayWwe>**





## Physically-Based Modeling (PBM) [2]: Applications in Movies & Games

star wars Podrace in HD

natzi1111a1 10 videos



Star Wars Episode I: The Phantom Menace © 1999  
Lucasfilm, Inc. <http://youtu.be/d4PSMXUCi-0>





## Summary

- Reading for Today: §5.3, Eberly 2<sup>e</sup>; **CGA Handout**
- Reading for Next Class: Chapter 14, Eberly 2<sup>e</sup>
- Last Class: Lab on Particle Systems; Dissection of Working Program
- Today: Computer-Generated Animation Concluded
  - \* **CGA of autonomous agents (robots, swarms) vs. animation by hand**
  - \* **Degrees of freedom (DOFs) and kinds of joints**
  - \* **Forward kinematics (FK)**
    - Forward problem illustrated
    - Control problem
  - \* **Inverse kinematics (IK)**
    - IK (finding angles) vs. mechanical problem of finding forces
    - Analytical models
    - Iterative models (Jacobian-based)
  - \* **Ragdoll physics**
- Next Class: Ray Tracing, Part 1 of 2





## Terminology

- **Emitter** – Point, Line, Plane or Region from which Particles Originate
- **Particle Fountain** – Particle System with Directional Emitter
- **Sprite** (Wikipedia: <http://bit.ly/gylnPg>)
  - \* **Definition:** 2-D image or animation made part of larger scene
  - \* **Point sprite** (Saar & Rotzler, 2008): <http://bit.ly/fkjBPY>
- **Joints:** Parts of Robot / Articulated Figure That Turn, Slide
  - \* **Revolute:** able to turn (rotate), forming angle between bones
  - \* **Prismatic (aka slider):** “bone” slides through – <http://bit.ly/hScjoe>
  - \* **Spherical (aka ball joint):** “bone” rotates around socket
  - \* **Cylindrical (aka hinge):** flaps wrap around joint, joined to surfaces
- **Effectors:** Parts of Robot / Articulated Figure That Act (e.g., Hand, Foot)
- **Bones:** Effectors, Other Parts That Rotate about, Slide through Joints
- **Procedural Animation:** Automatic Generation of Motion *via* Simulation
  - \* **Ragdoll physics:** procedural animation for inert characters
  - \* **Other types:** particle systems, *N*-body dynamics

